# 7-7 对称性的利用



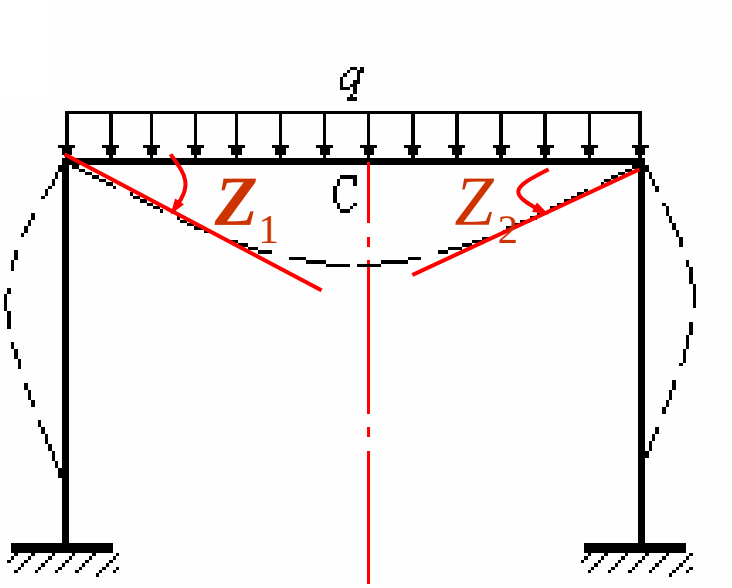
位移法中对称性的利用关键是半结构的选取

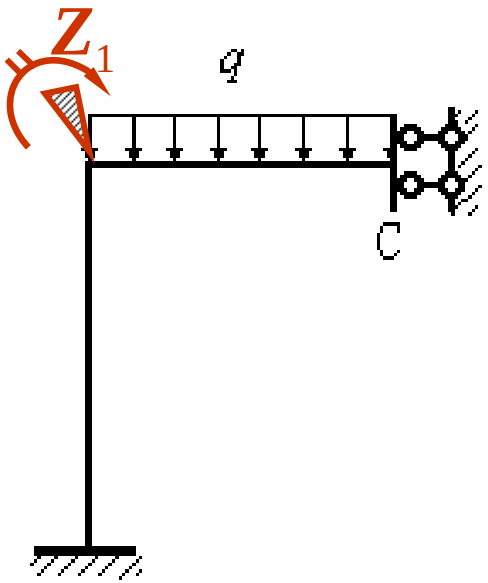


# 奇数跨对称结构



**位移法中对称性的利用关键是半结构的选取**

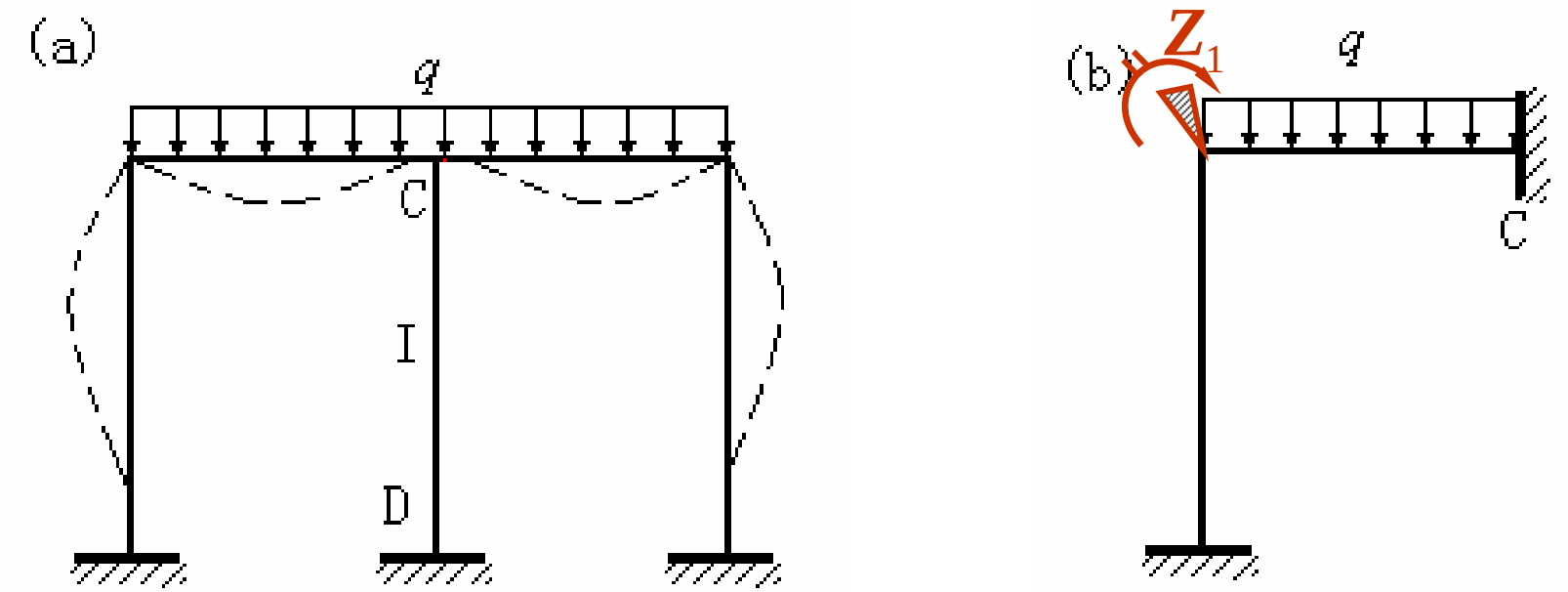



$$Z_1 \neq Z_2 \qquad Z_1 = -Z_2$$

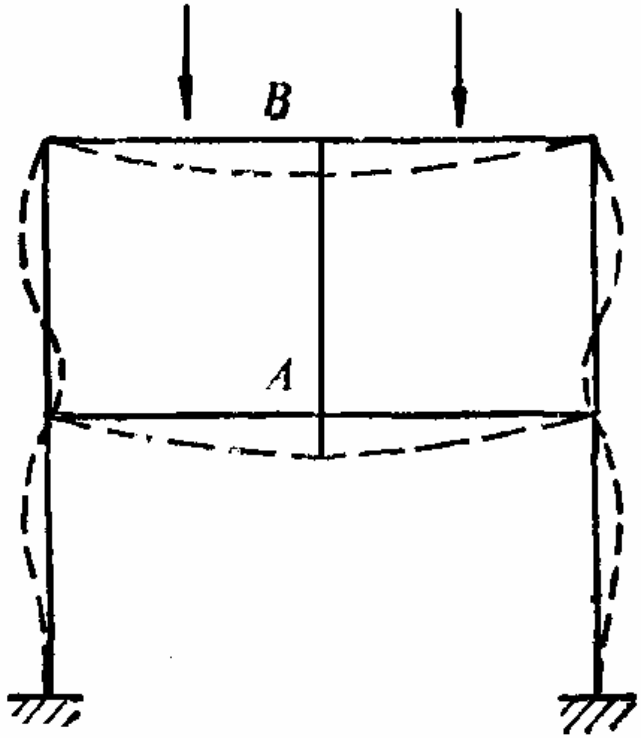
# 偶数跨对称结构

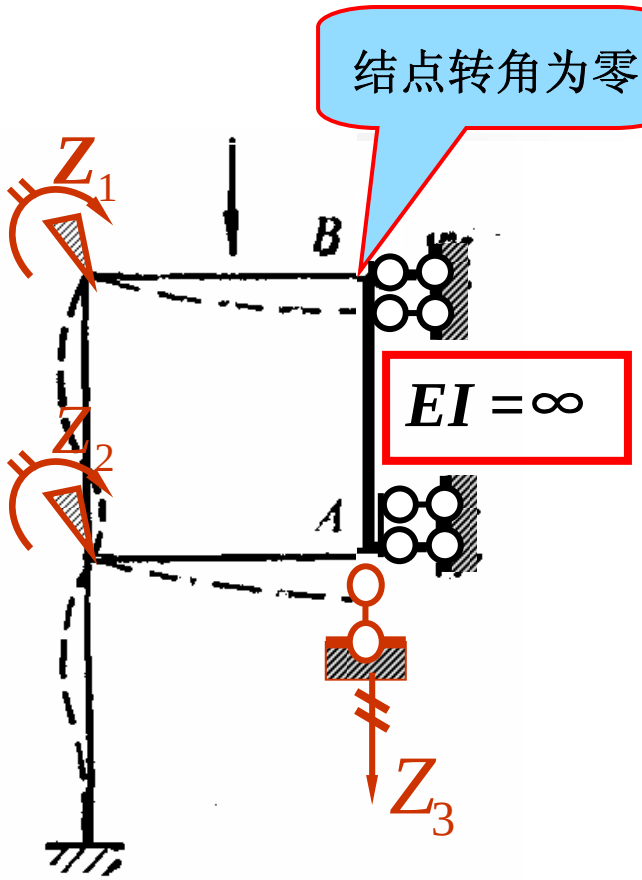




# 偶数跨对称结构





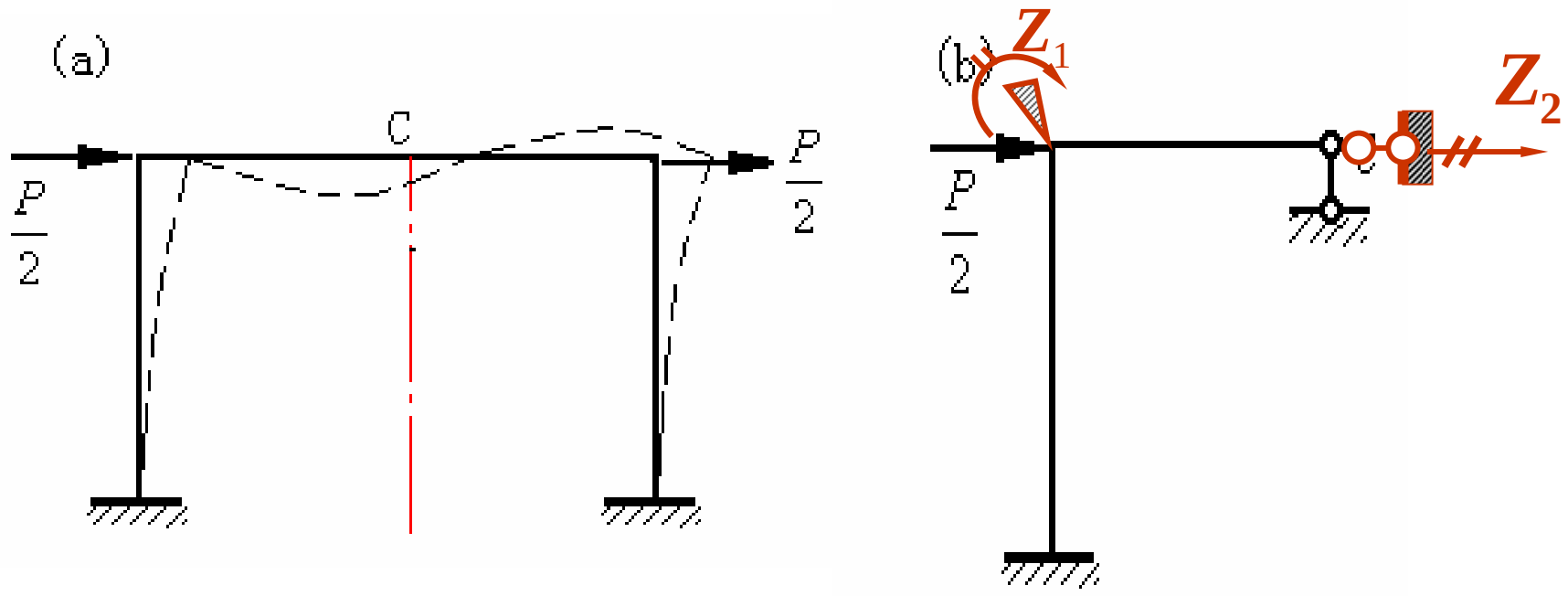

在对称轴上的结点***B*** 和***A*** 均无转角及水平线位移，但可发生竖向线位移且两点相等，中央竖杆***AB***不发生挠曲。

截取半结构时，可将杆***AB*** 看作刚性杆而保留，并在结点***B***、***A*** 分别加上水平链杆支承。

# 奇数跨对称结构

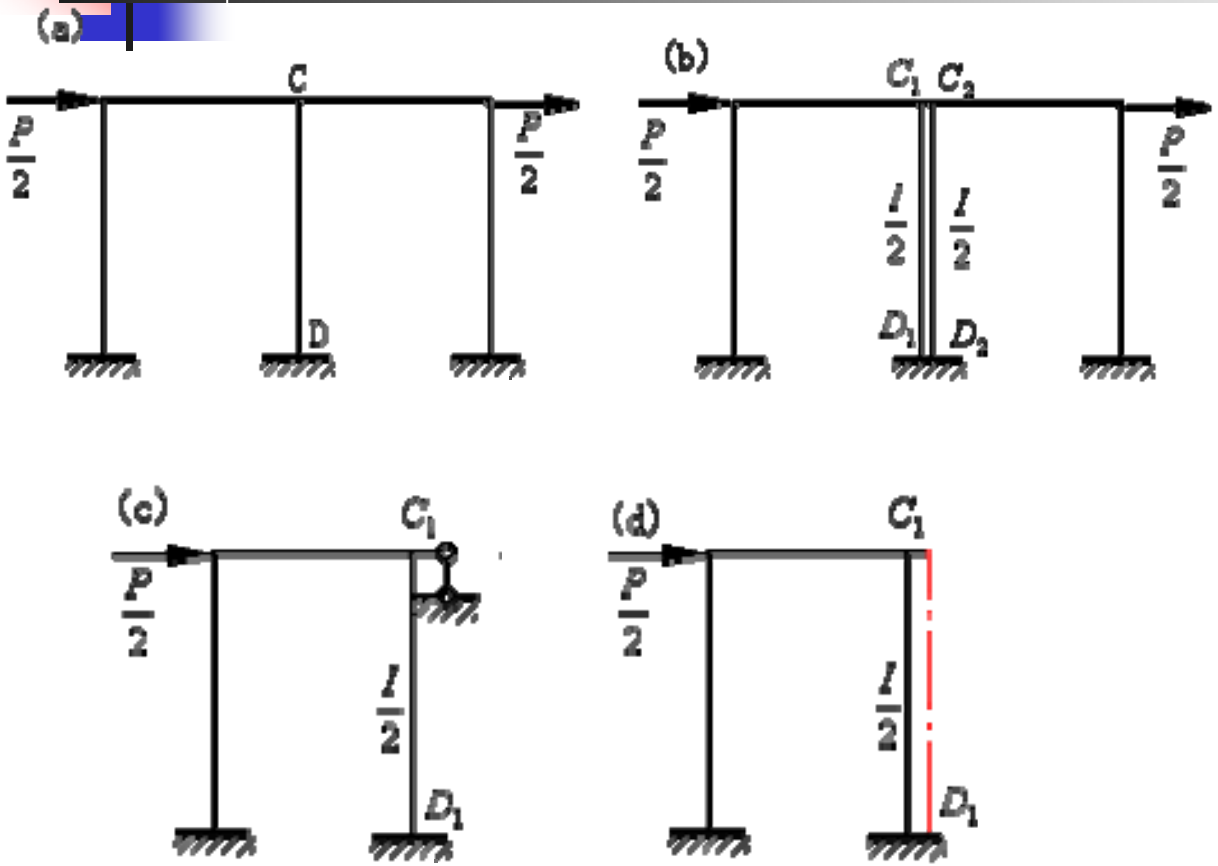
## （2）反对称荷载

在对称轴上的截面*C*没有竖向位移，但可有转角和水平位移。



# 偶数跨对称结构

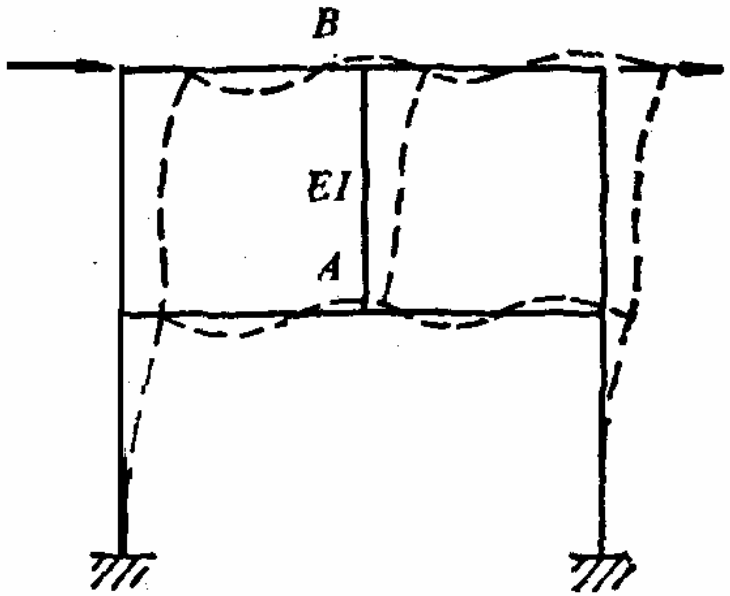


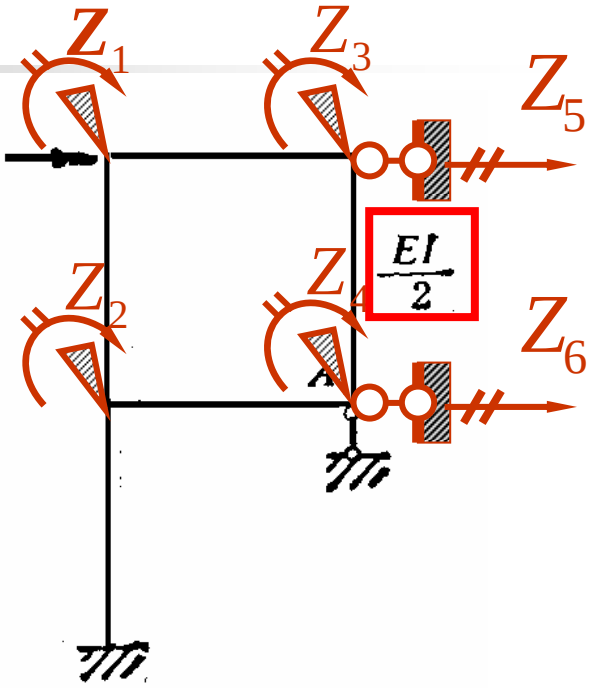



在对称轴上，柱*CD*没有轴力和轴向位移,但有弯矩和弯曲变形。可将中间柱分成两根柱，分柱的抗弯刚度为原柱的一半。

因为忽略轴向变形的影响，*C*处的竖向支杆可取消。

# 偶数跨对称结构

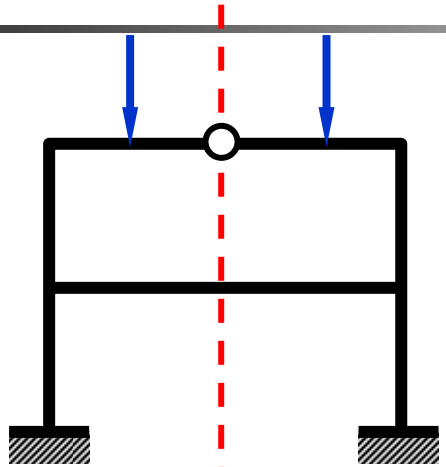


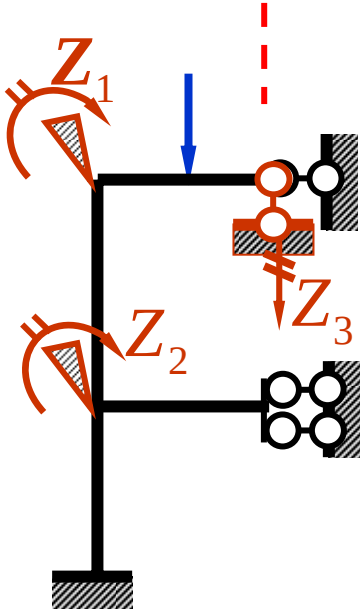

对称轴上的结点***A*** 和***B*** 均有转角和侧移，但无竖向线位移，中央竖杆***AB***发生挠曲变形。

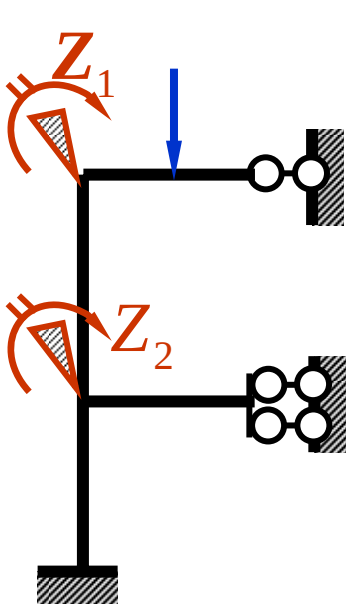
在截取半结构计算时，除了取竖杆***AB*** 刚度之半(***EI***/2)外，还应在***A***处加一竖向链杆支承。

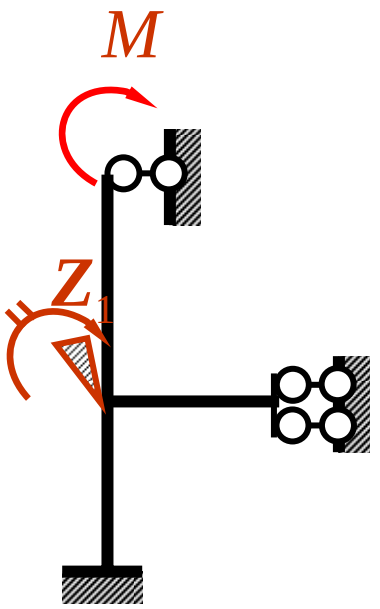
# 讨论：



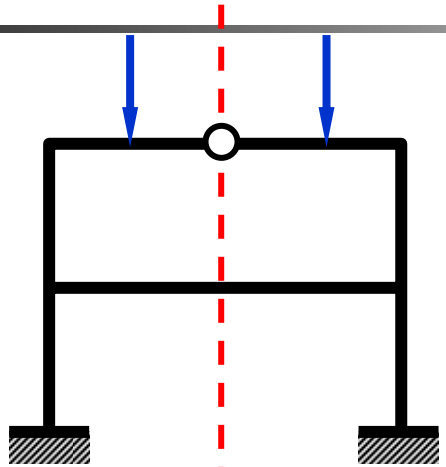
最少未知量

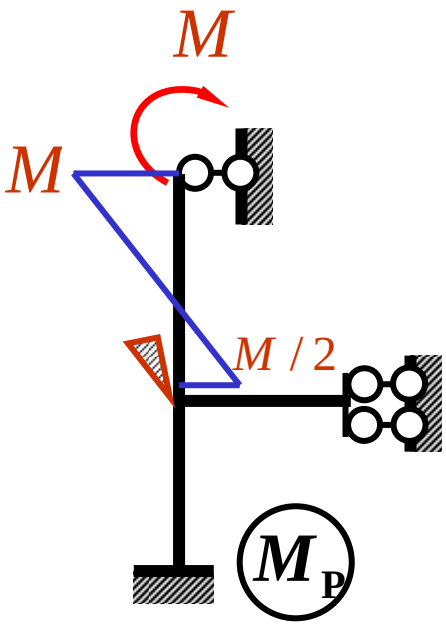


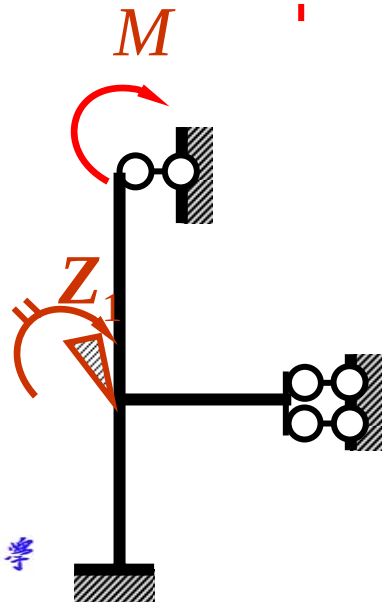




西南交通大学

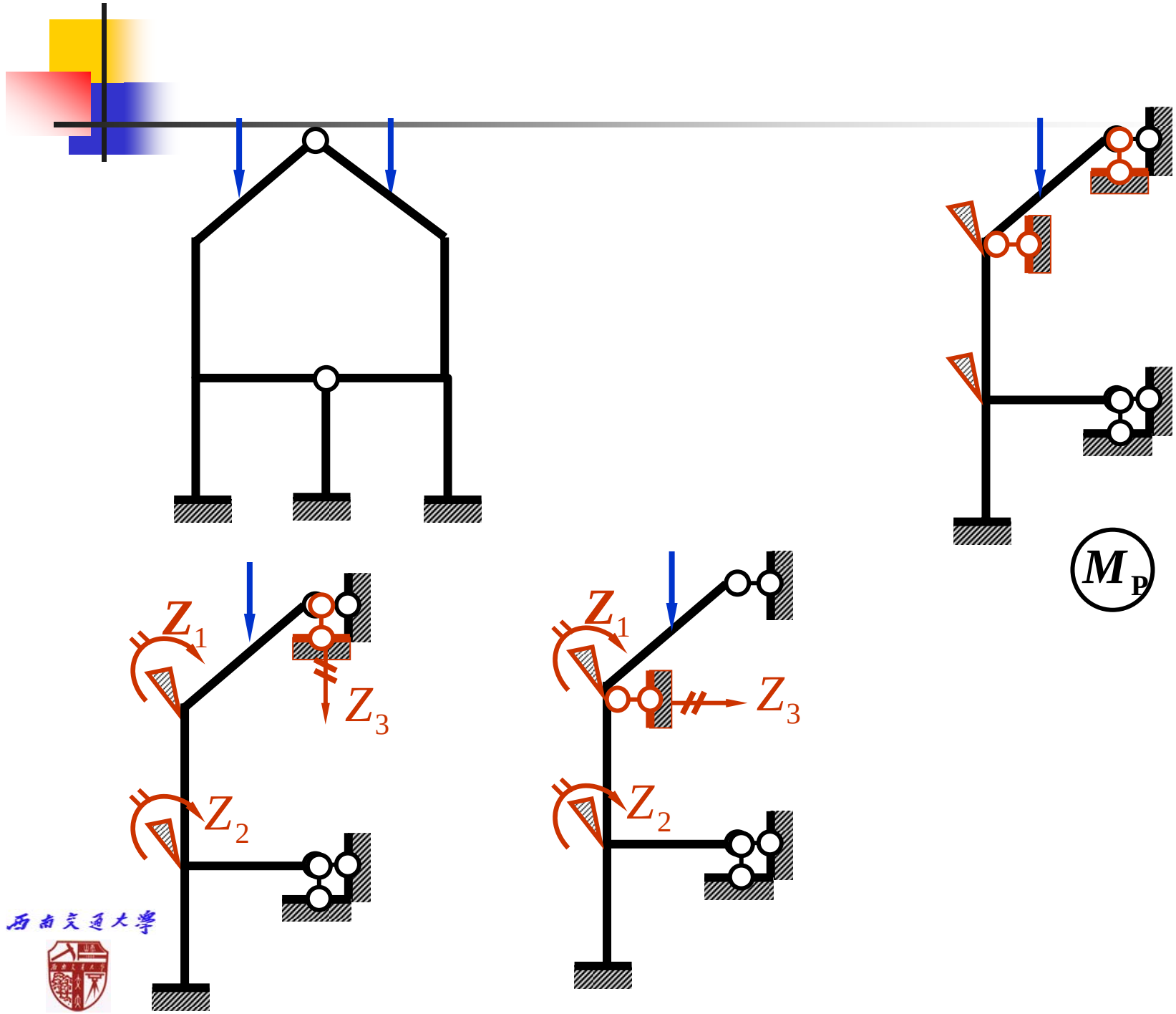


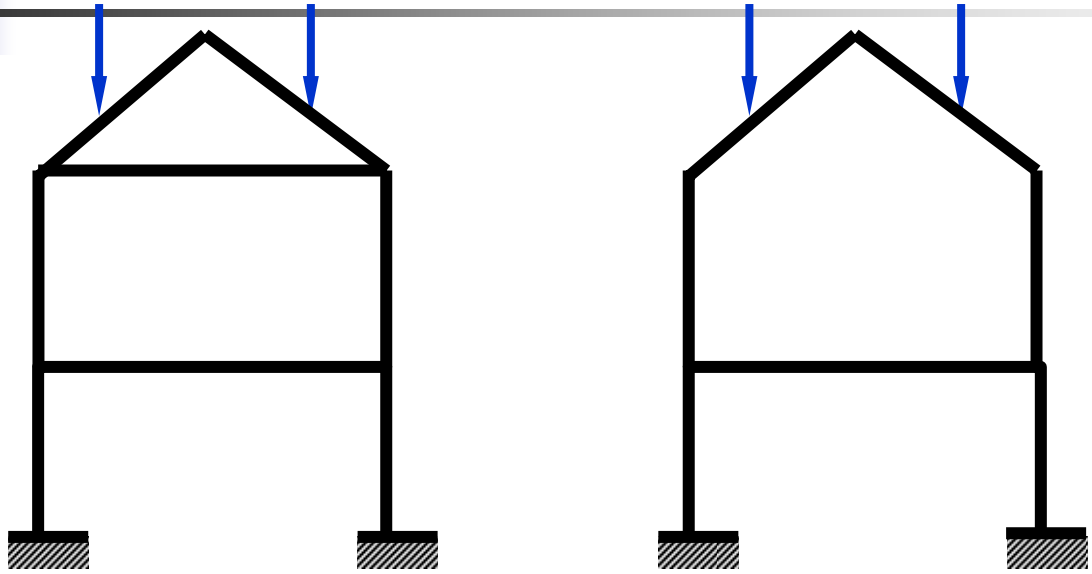


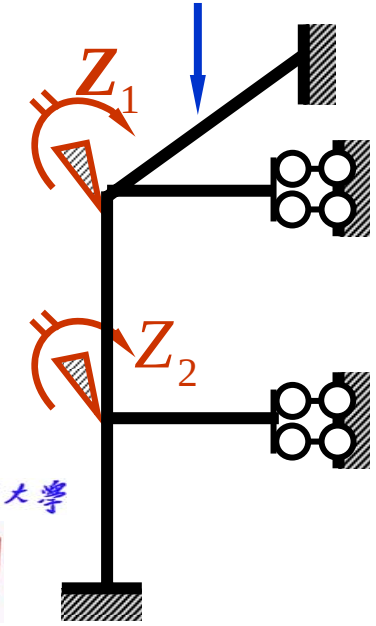



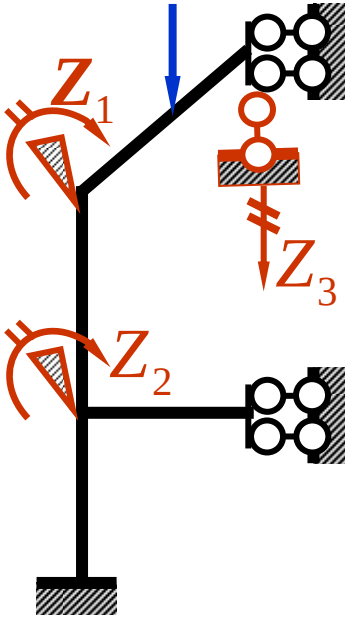

$$r_{11}Z_1 + R_{1P} = 0$$

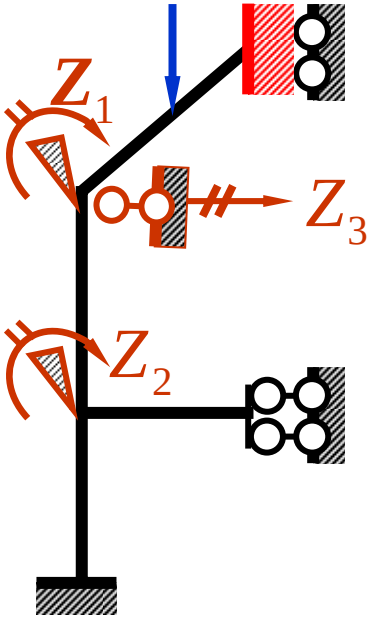








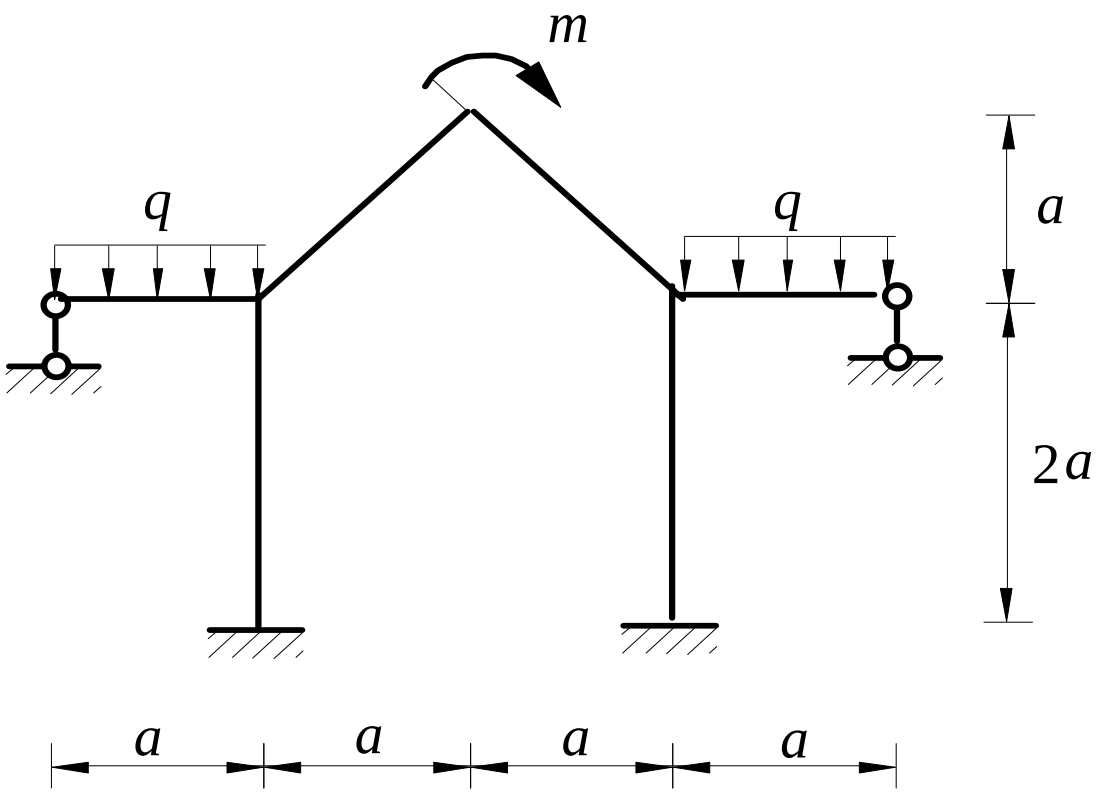




西南交通大學

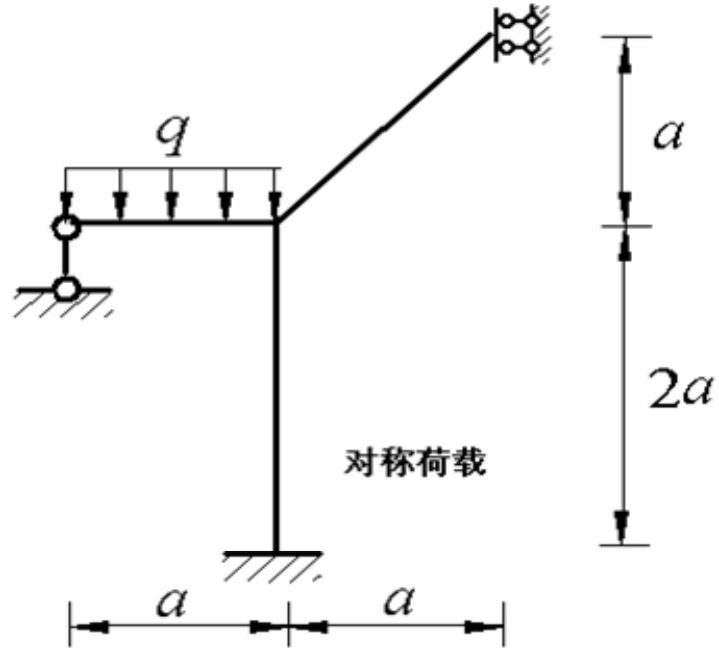
# 例

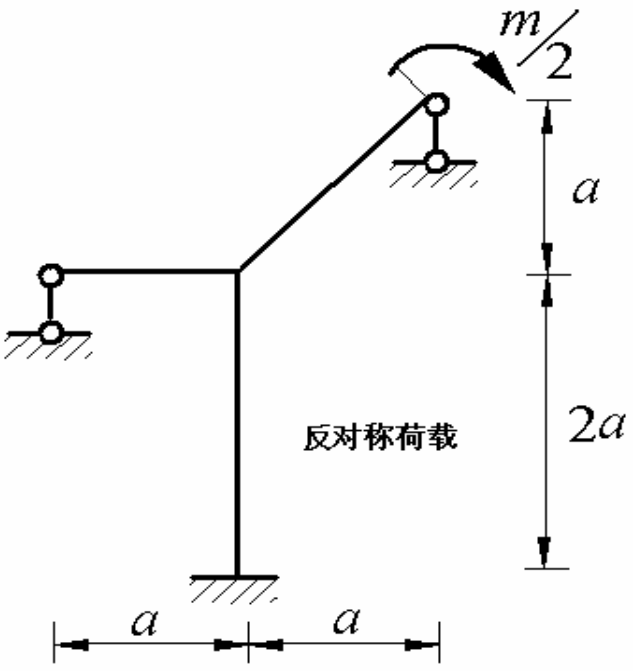
列出用位移法并利用对称性计算图示刚架的基本结构及典型方程。（各杆的 $EI$ =常数）

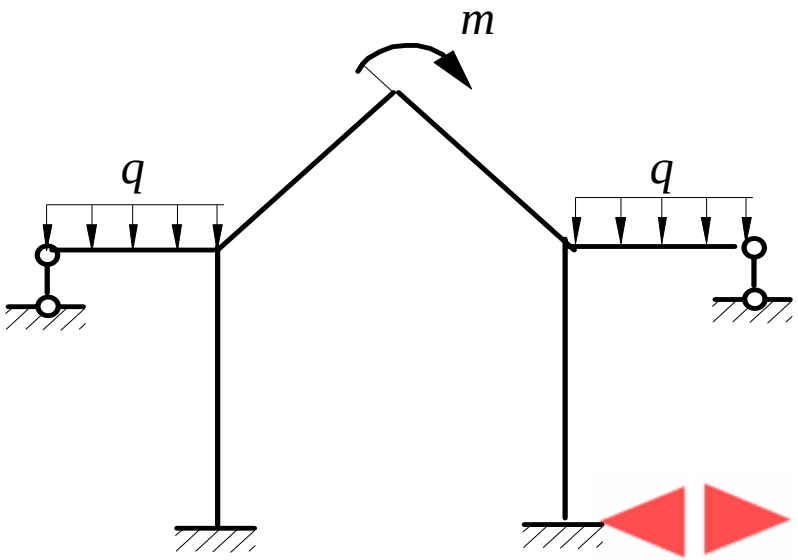


# 取半结构



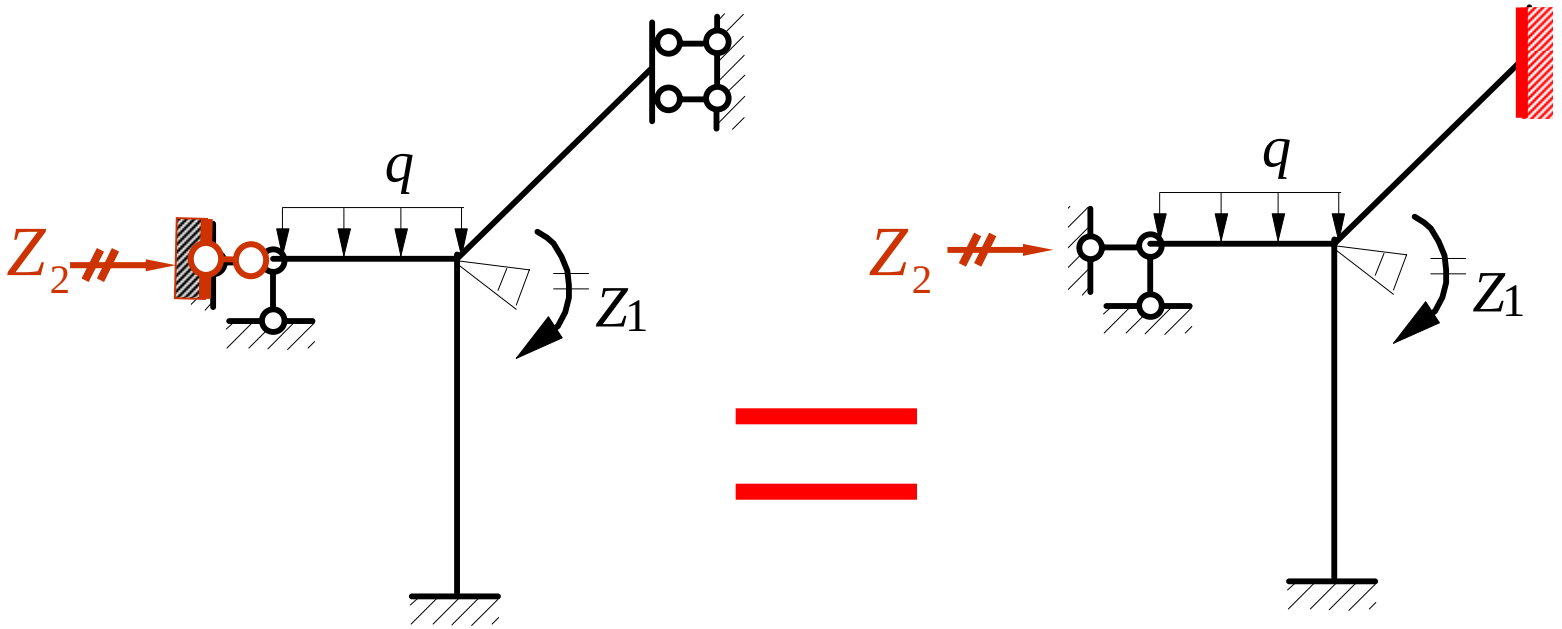


# 对称荷载作用下基本结构



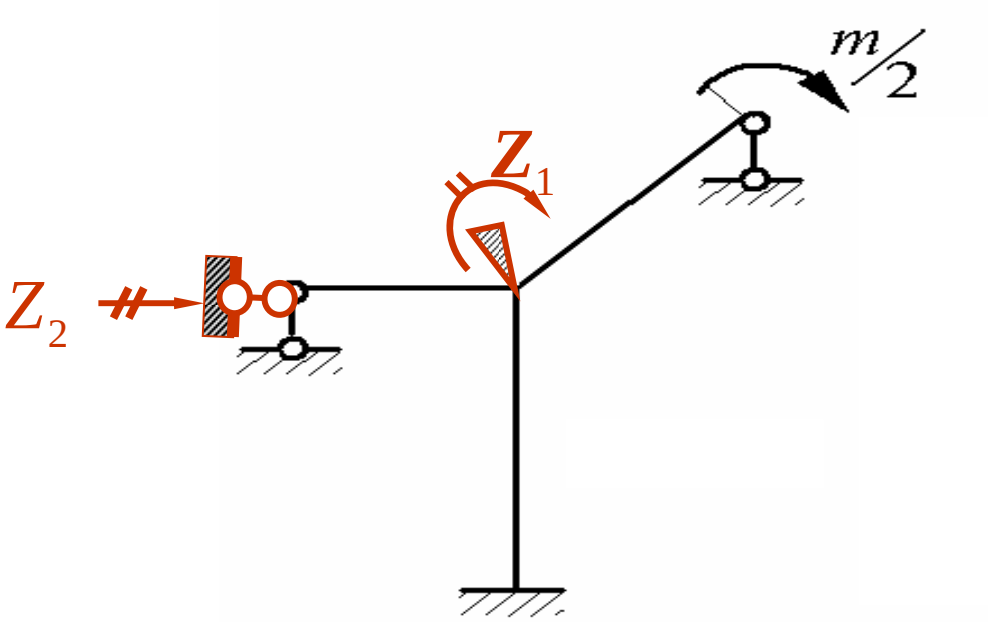



典型方程：

$$\left.\begin{aligned} r_{11}Z_1 + r_{12}Z_2 + R_{1P} = 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2P} = 0 \end{aligned}\right\}$$

# 反对称荷载作用下基本结构



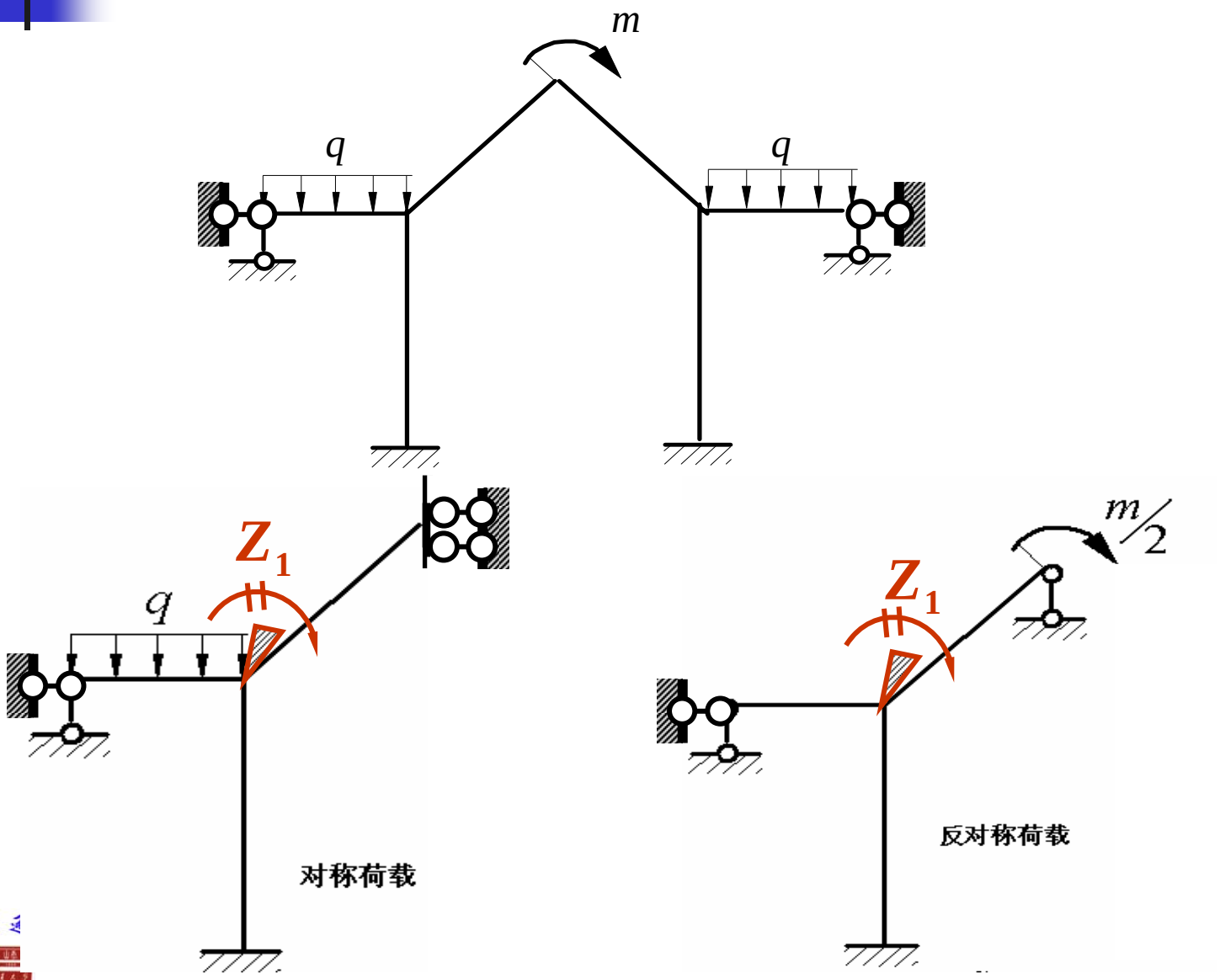



典型方程：

$$\left.\begin{aligned} r_{33}Z_3 + r_{34}Z_4 + R_{3P} = 0 \\ r_{43}Z_3 + r_{44}Z_4 + R_{4P} = 0 \end{aligned}\right\}$$

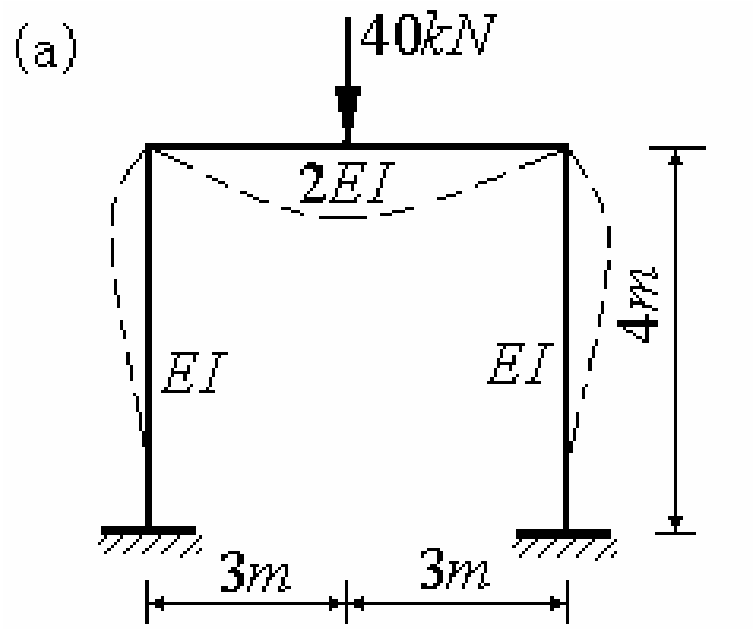
# 取半结构示例

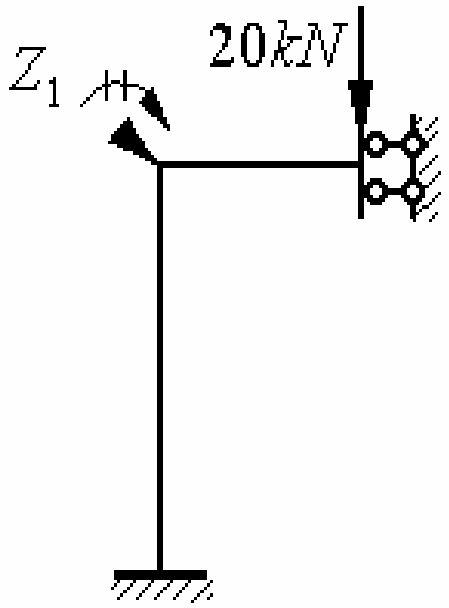




# 例1

利用对称性简化图a所示的对称结构，取出最简的计算简图、基本体系，并作出*M*图。

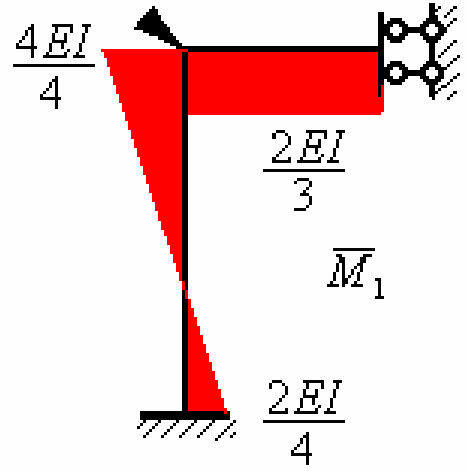


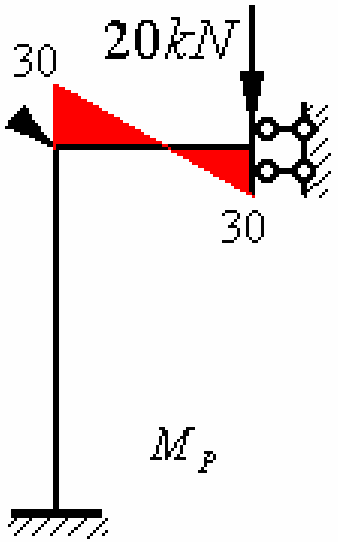




$$r_{11}Z_1 + R_{1P} = 0$$

# 最简的基本体系及M图





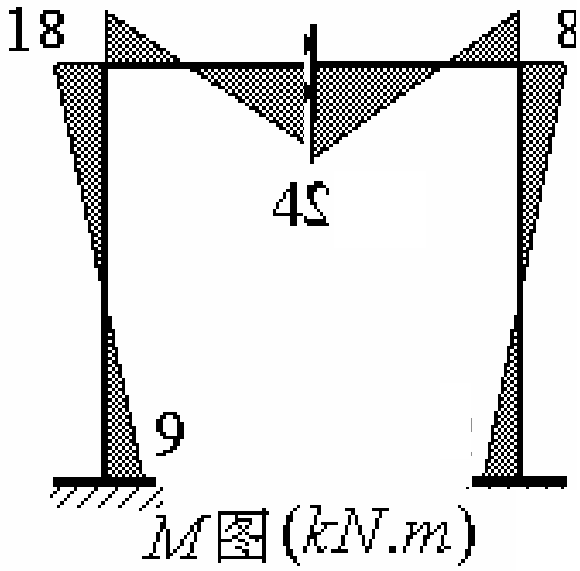




$$r_{11} = \frac{5EI}{3}$$

$$R_{1P} = -30kN \cdot m$$

$$Z_1 = \frac{90}{5EI}$$

$$M = \overline{M}_1 Z_1 + M_P$$

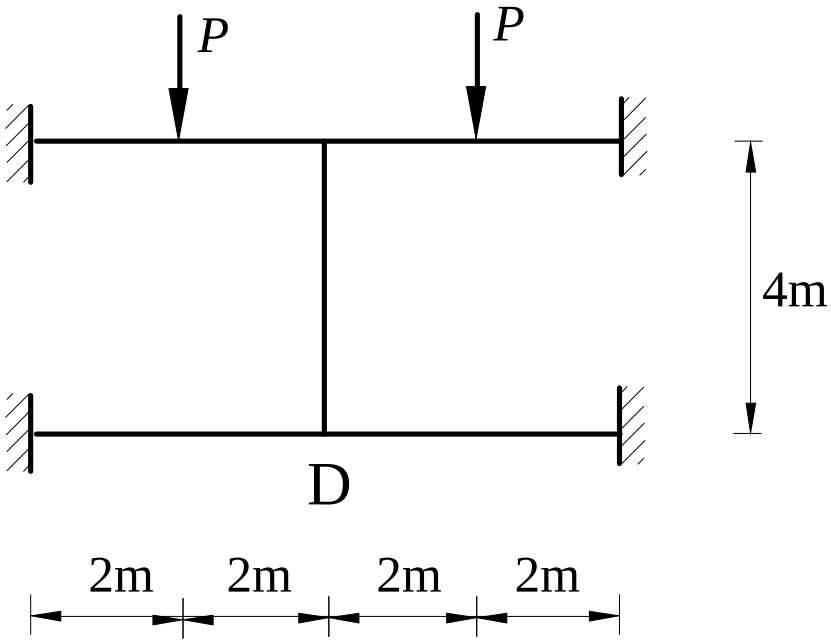



西南交通大学

# 例2

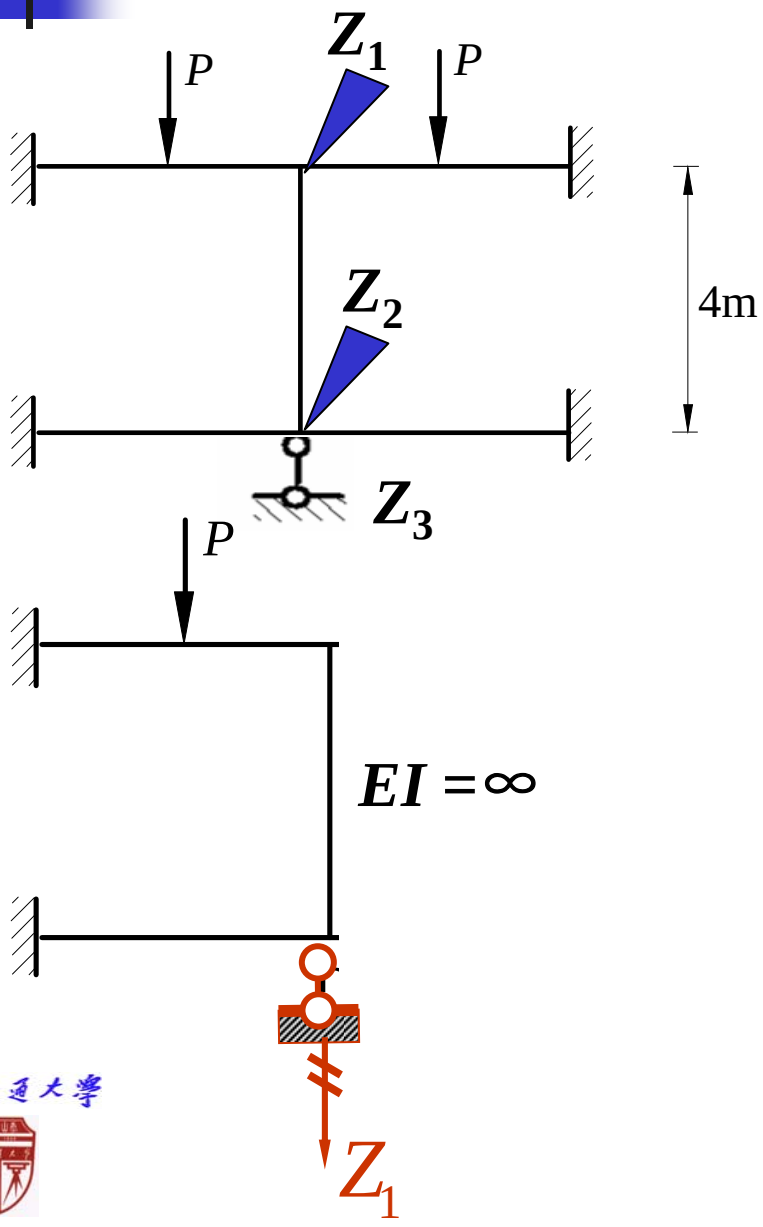


图示结构， 设$EI$=常数， $P$=10kN，试画出刚架的$M$ 图。



# 基本方程






由于结构对称，荷载对称

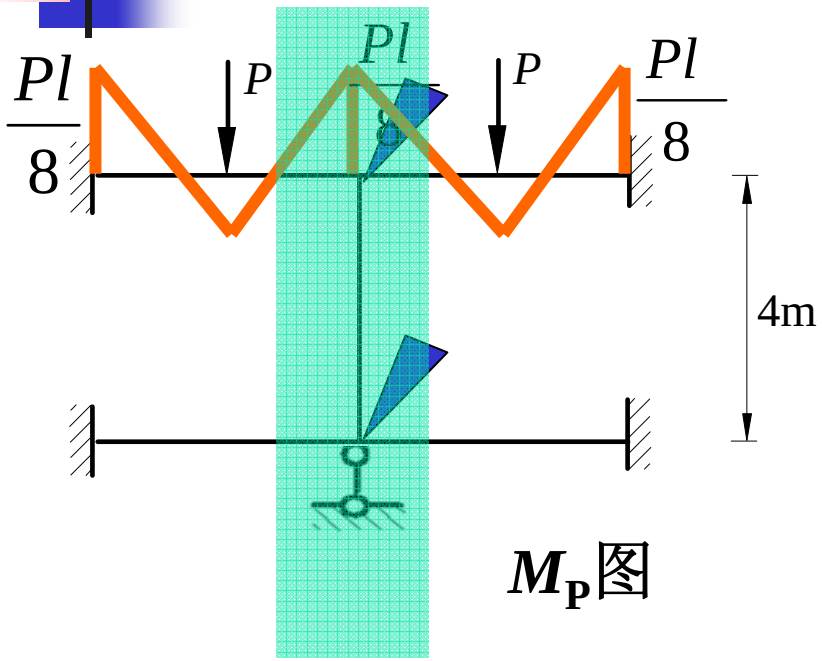
$Z_1 = Z_2 = 0$

$Z_3 = Y_D$

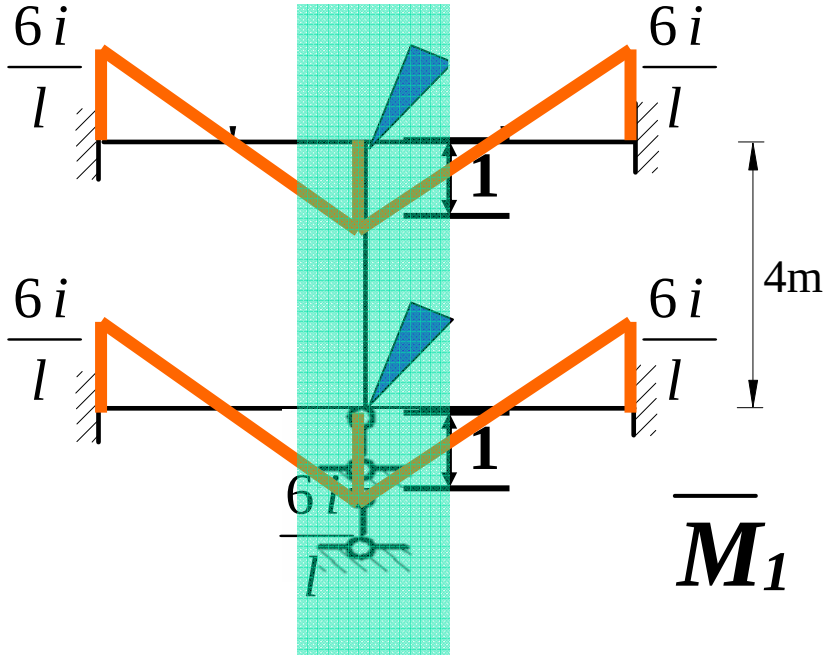
基本方程为：

$$r_{11}Z_1 + R_{1P} = 0$$

# 求系数及自由项、作出$M_1$、$M_P$图




$M_P$图




$\overline{M}_1$

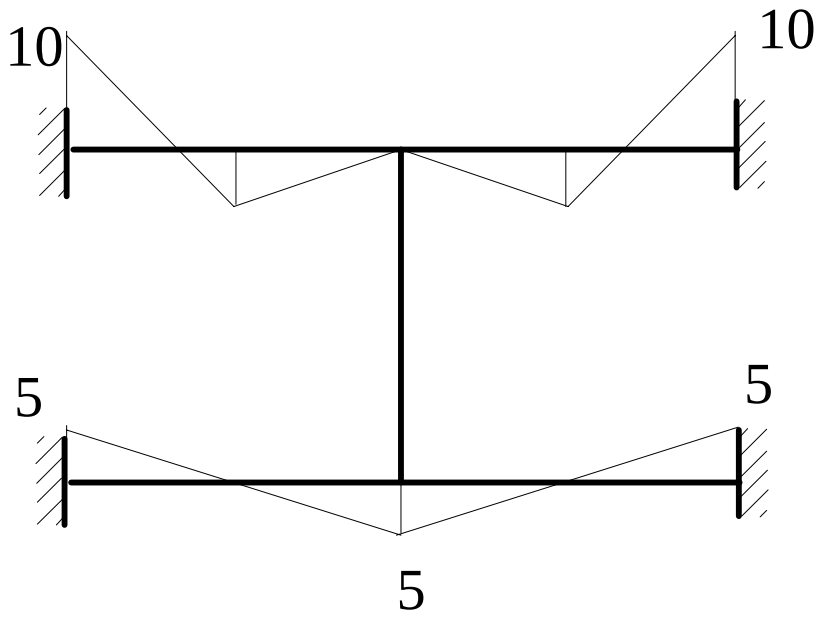
$$r_{11}=\frac{12i}{l^2}\times 4=\frac{48i}{l^2}=\frac{48i}{16}=3i$$

$$R_{1P}=-P$$

# 作出$M$图



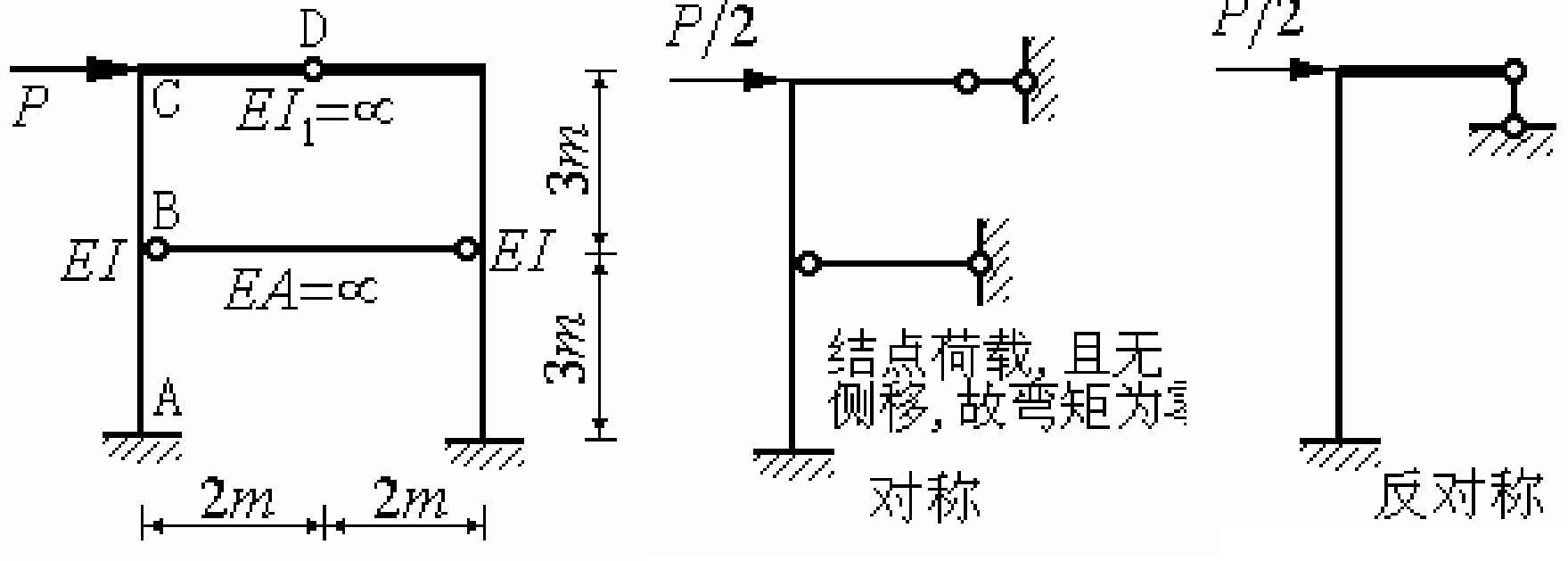
$$Z_1 = -\frac{R_{1P}}{r_{11}} = \frac{P}{3i} = 4P/(3EI)(\downarrow) \qquad M = \overline{M}_1 Z_1 + M_P$$




$M$ 图（ kN·m ）

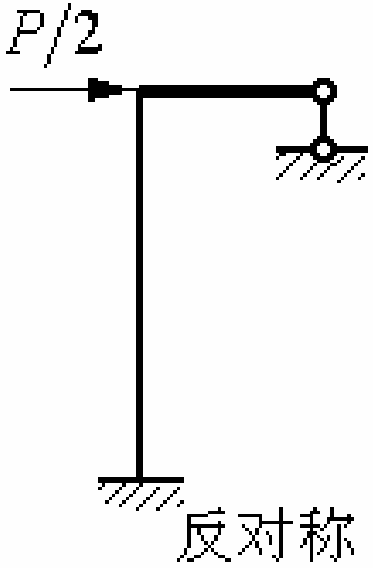
# 例3

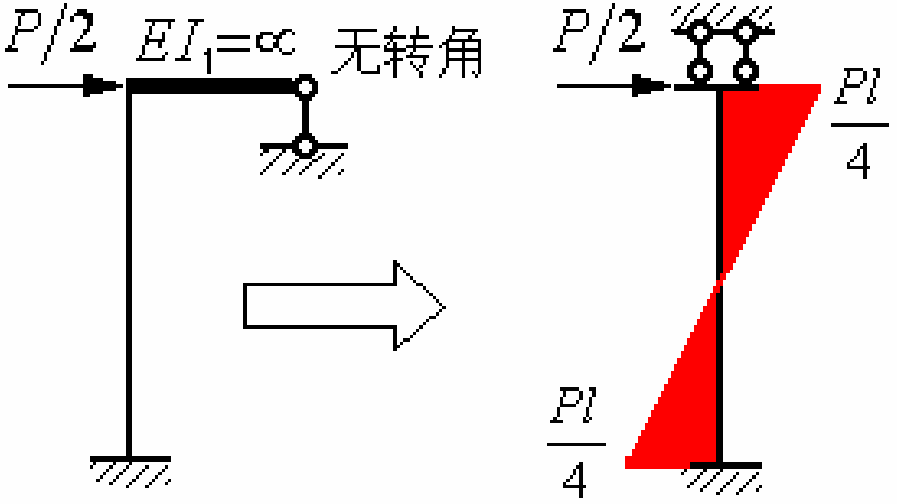
利用对称性简化图a所示的对称结构，取出最简的计算简图、基本体系，并作出*M*图。



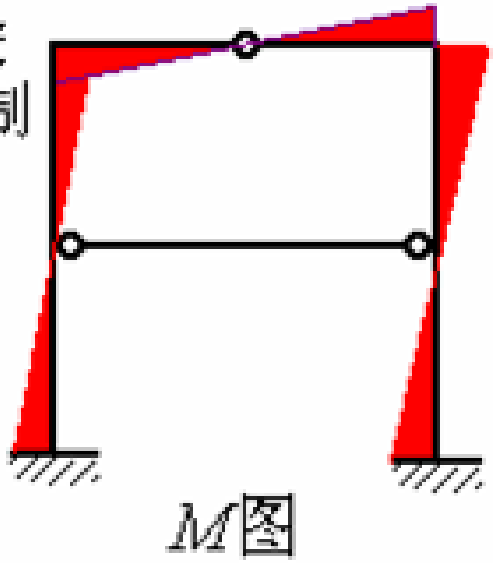
# 最简的计算简图及M图





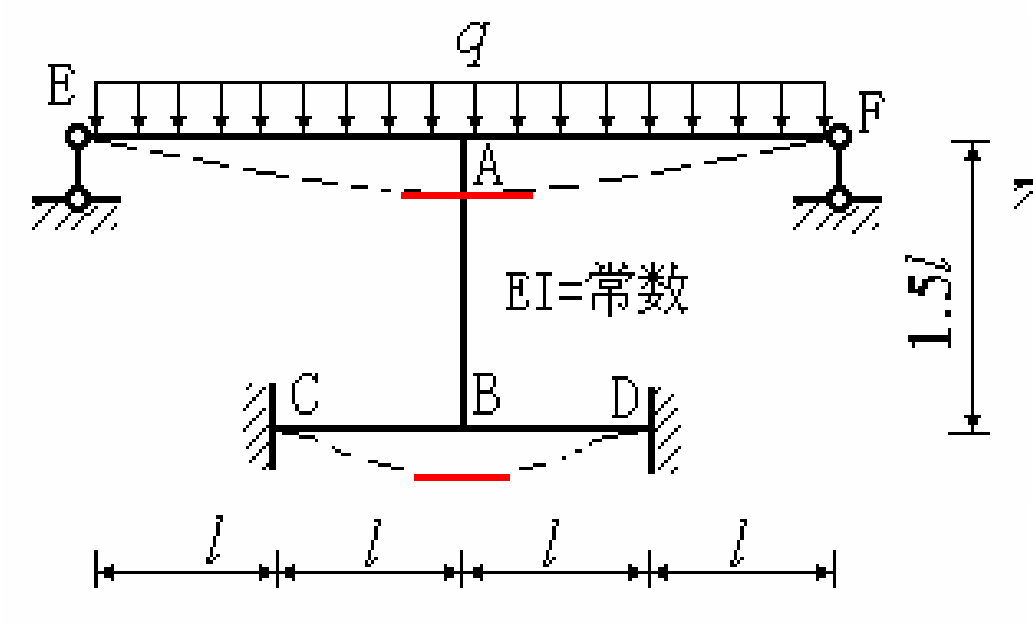



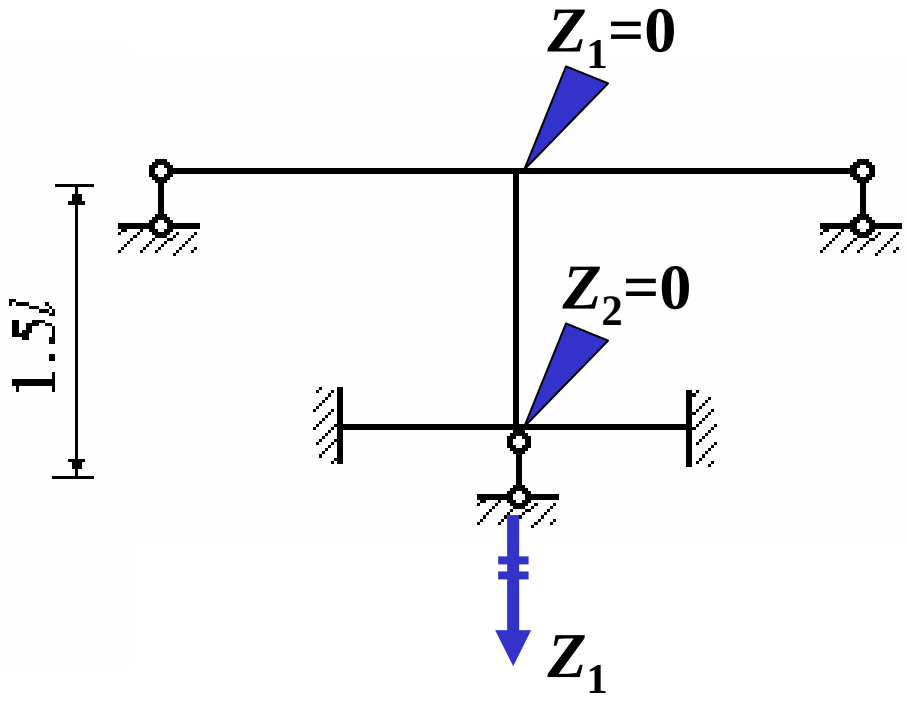

刚结点出按
平衡条件绘制



# 例4

利用对称性简化图示的对称结构，取出最简的计算简图、基本体系。

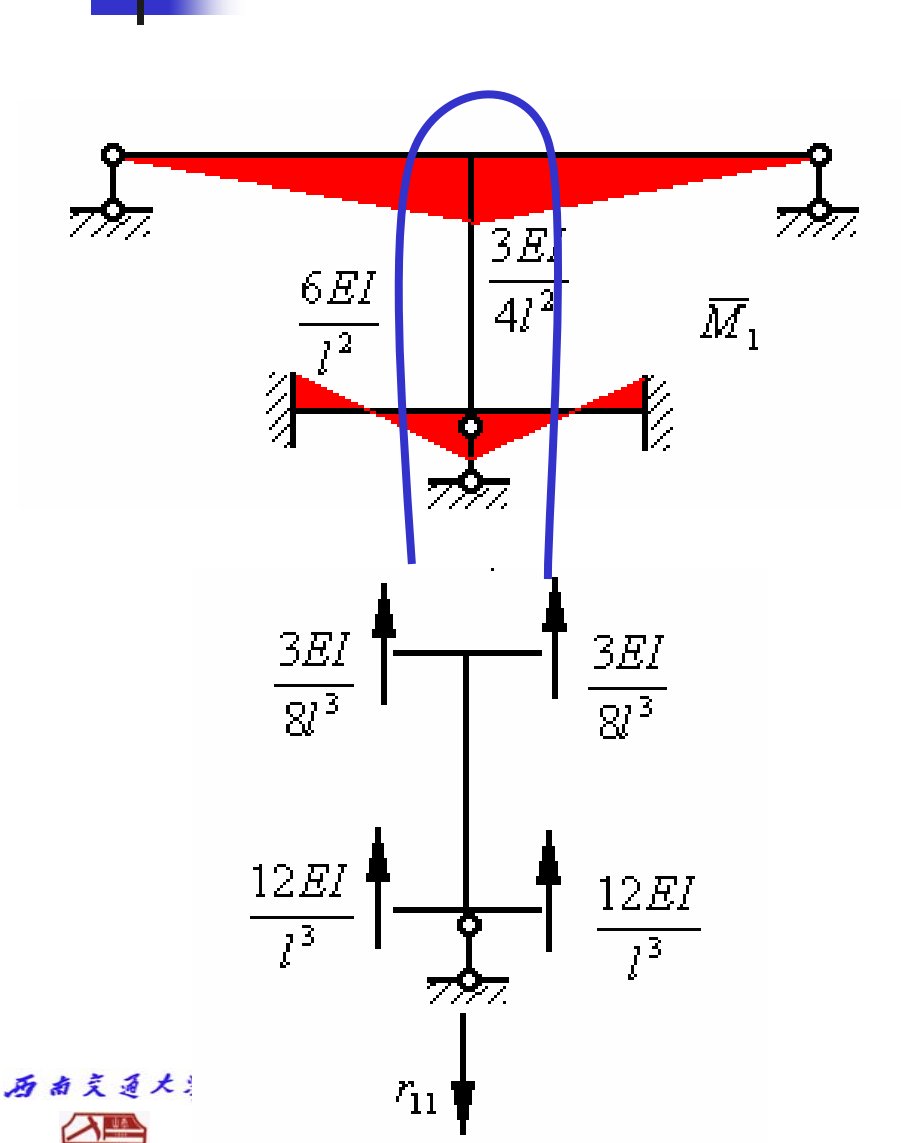


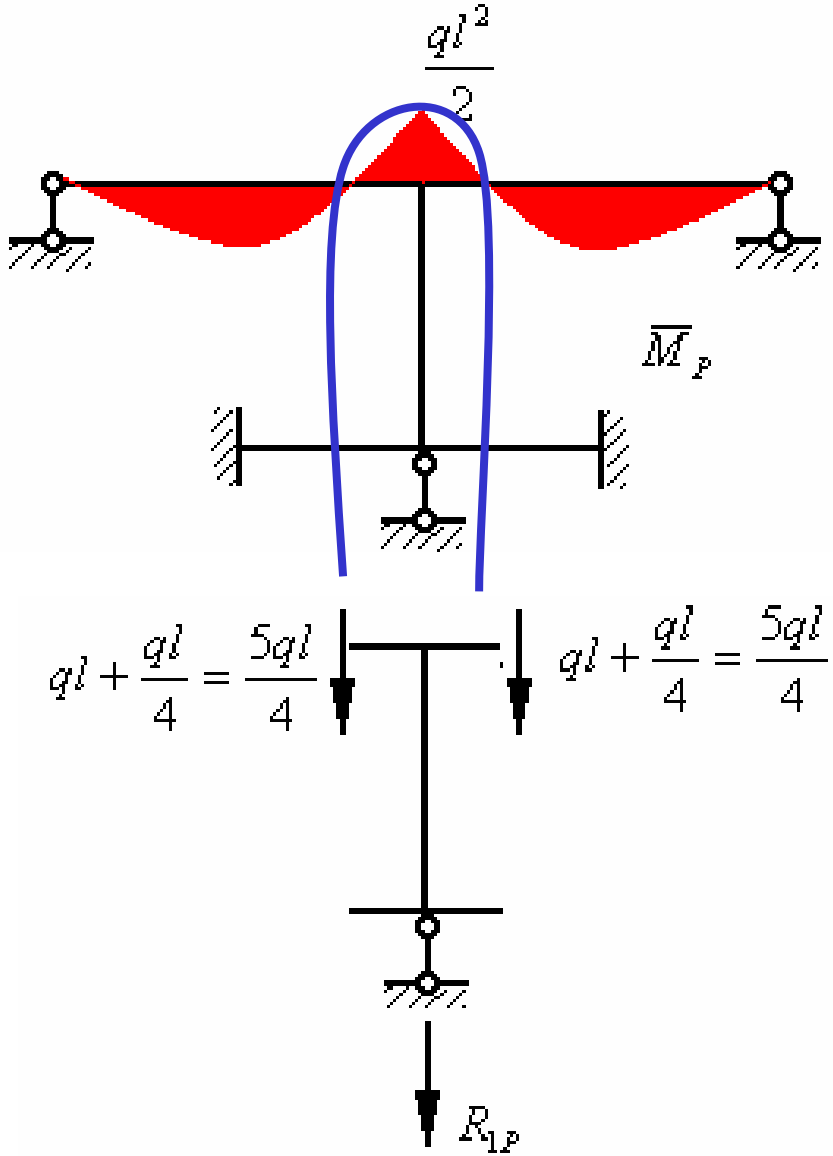




$$r_{11}Z_1 + R_{1P} = 0$$

# 系数和自由项的计算

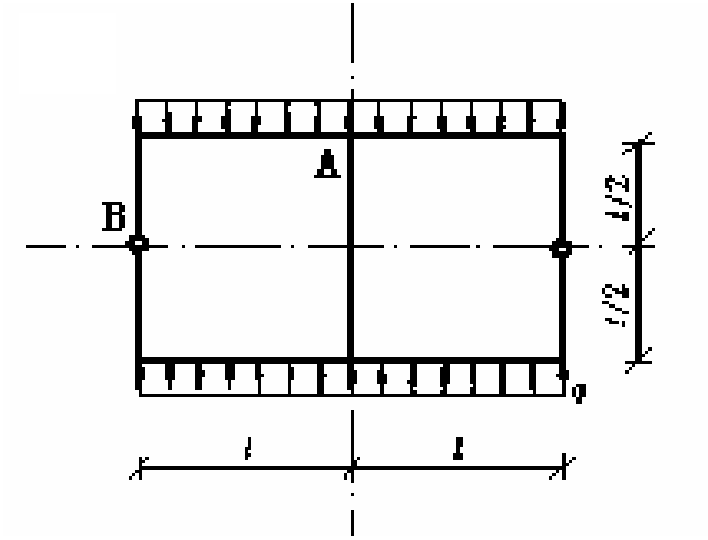


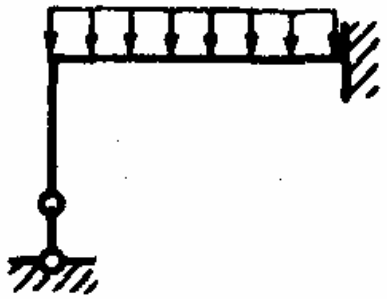

$\frac{6EI}{l^2}$
$\frac{3EI}{4l^2}$
$\overline{M}_1$
$\frac{3EI}{8l^3}$
$\frac{3EI}{8l^3}$
$\frac{12EI}{l^3}$
$\frac{12EI}{l^3}$
$r_{11}$

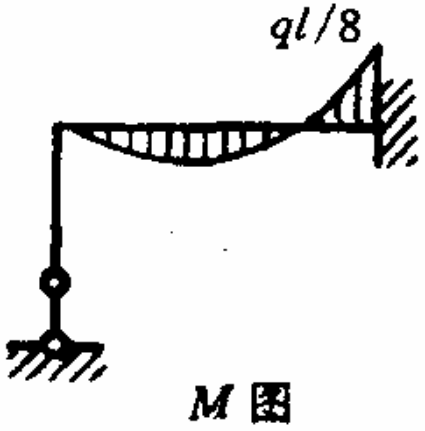

$\frac{ql^2}{2}$
$\overline{M}_P$
$ql+\frac{ql}{4}=\frac{5ql}{4}$
$ql+\frac{ql}{4}=\frac{5ql}{4}$
$R_{1P}$

# 例5

利用对称性简化图示的对称结构，取出最简的计算简图、基本体系，并作出*M*图。



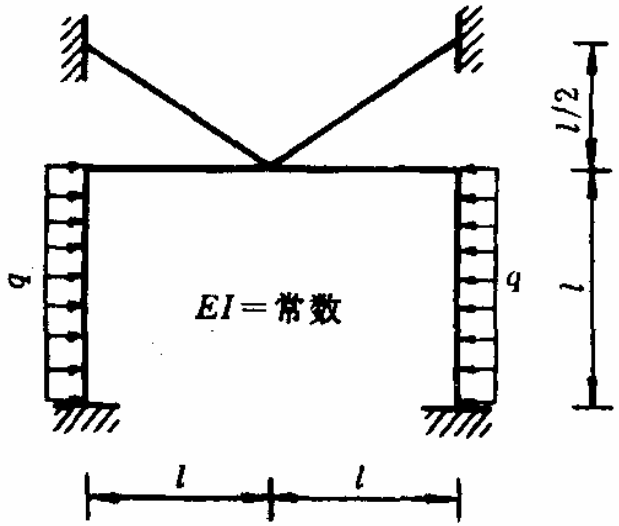



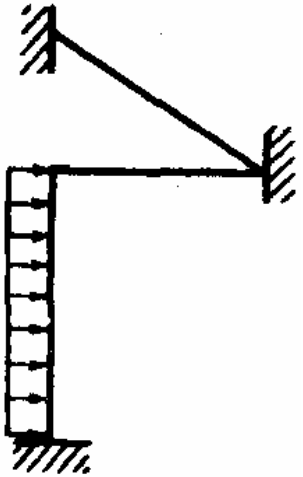


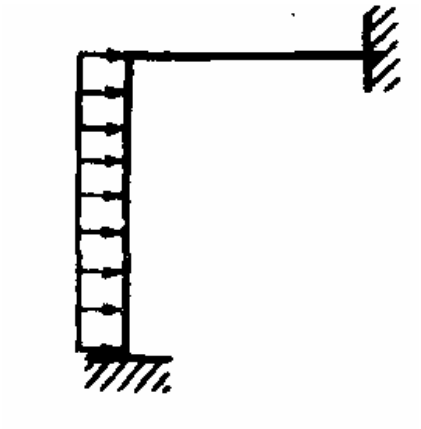

M 图

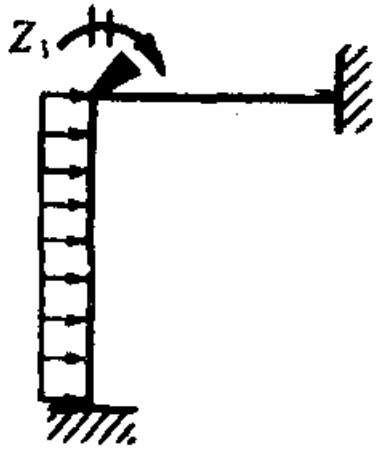
# 例6

利用对称性简化图示的结构，取出最简的计算简图及基本体系。



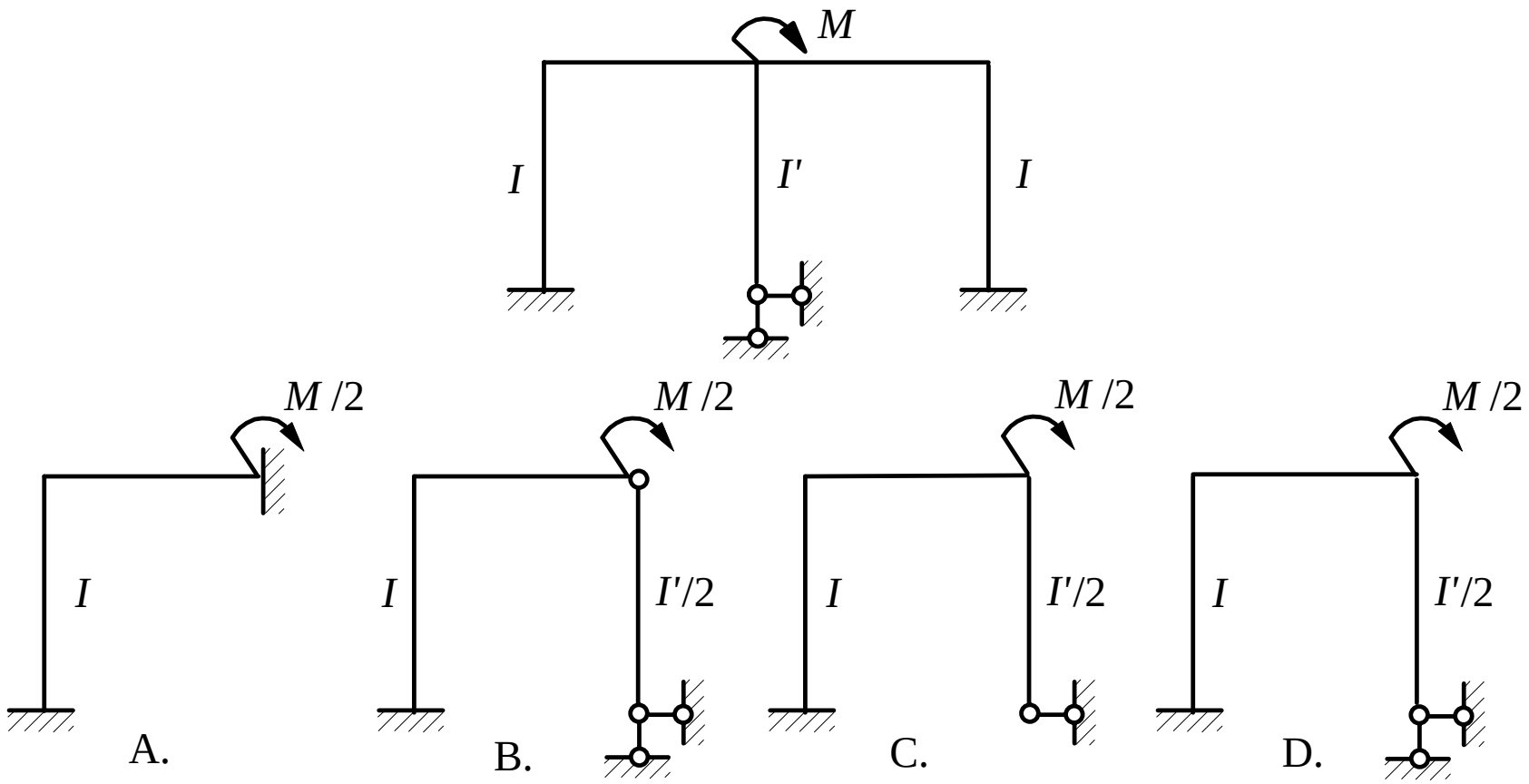







# 思考：

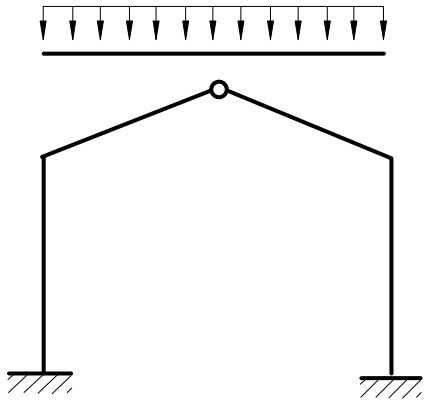
图示结构利用对称性的半结构为：




( D )

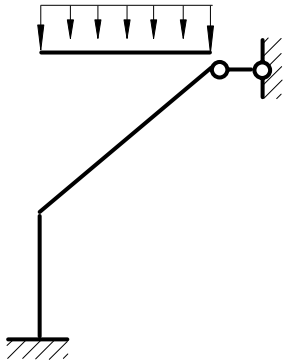
# 思考：

图示对称结构，其半结构计算简图为图：

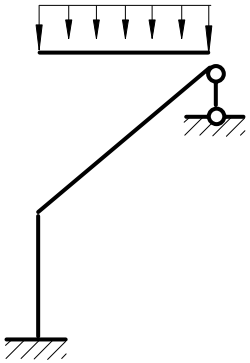


原 图

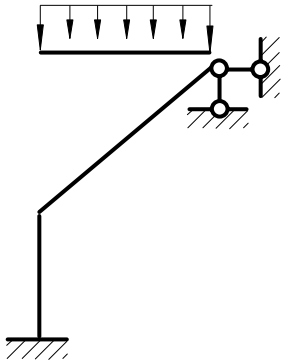
A.



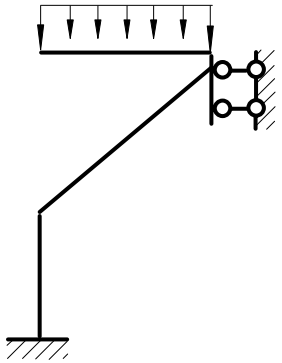
B.



C.



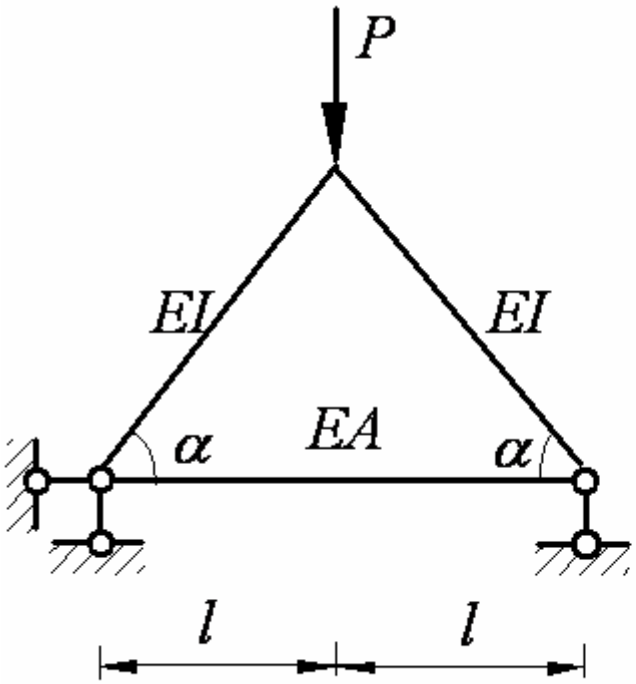
D.

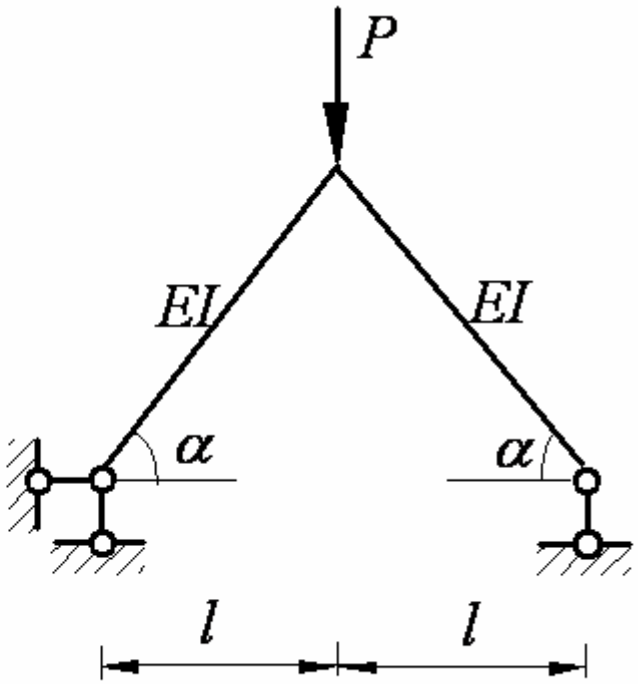


( A )

# 思考：

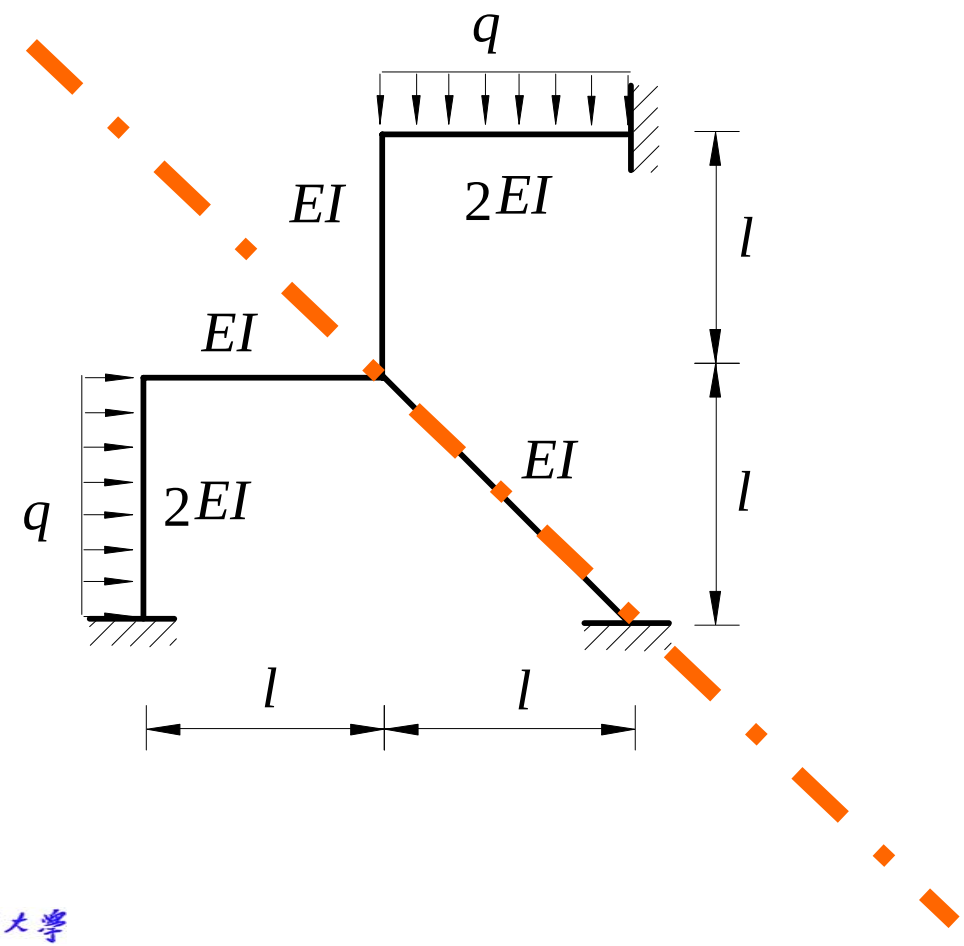
计算图*a*结构时，可简化为图*b*计算的条件是 _____

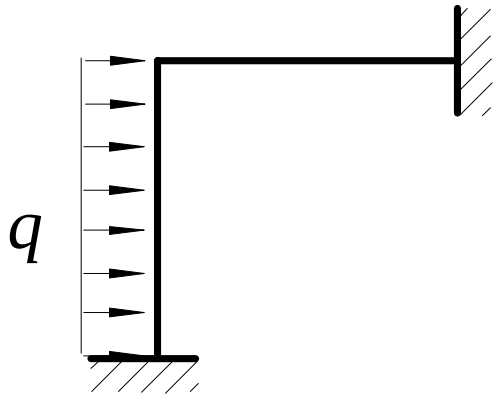


# 例 8

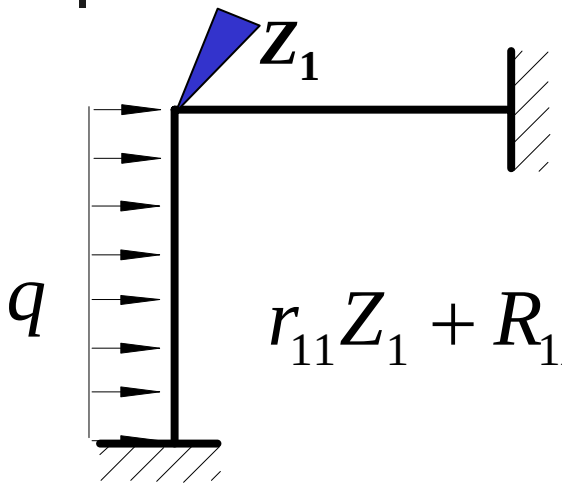
用位移法作图示结构的 $M$ 图。



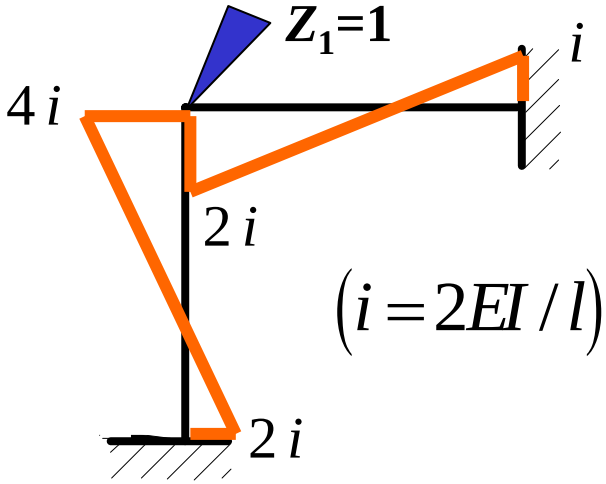



# 利用对称性，取半结构

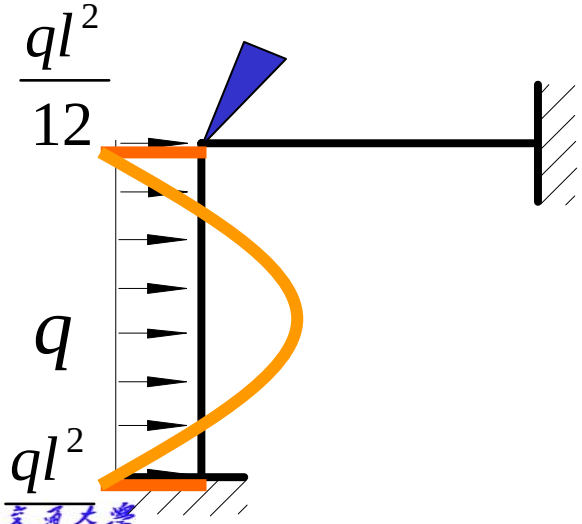





$$r_{11}Z_1 + R_{1P} = 0$$




$$(i = 2EI/l)$$

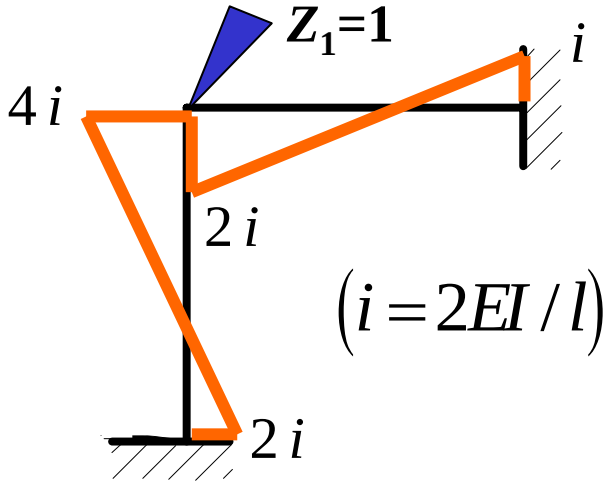



$$r_{11} = 6i \qquad R_{1P} = \frac{ql^2}{12}$$

$$Z_1 = -\frac{ql^2}{72i}$$

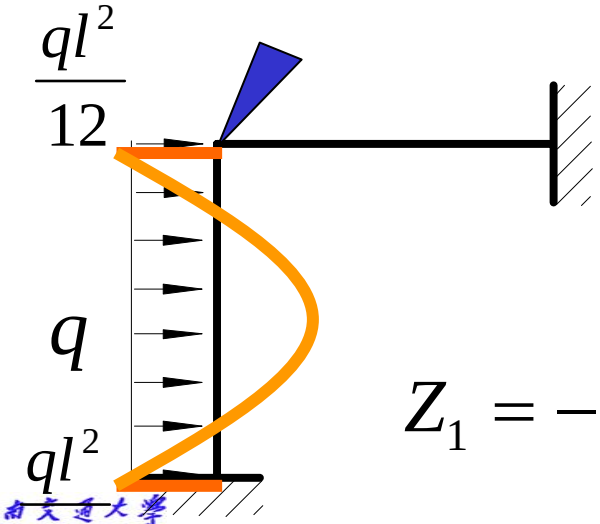
# 利用对称性，取半结构



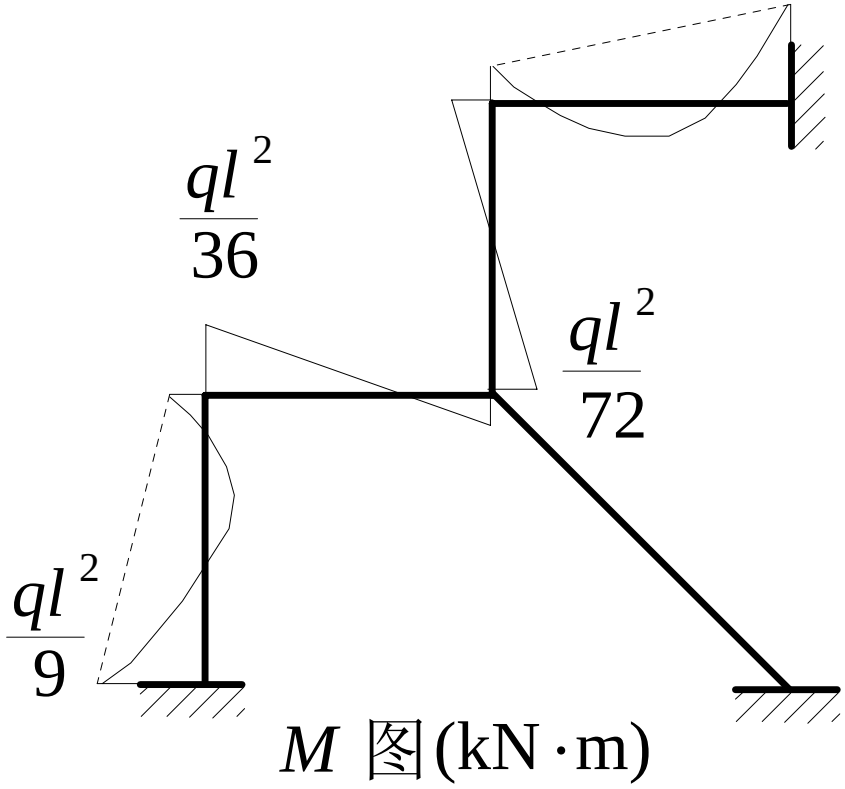



$$\left(i=2EI/l\right)$$

$$M=Z_1\overline{M}_1+M_P$$



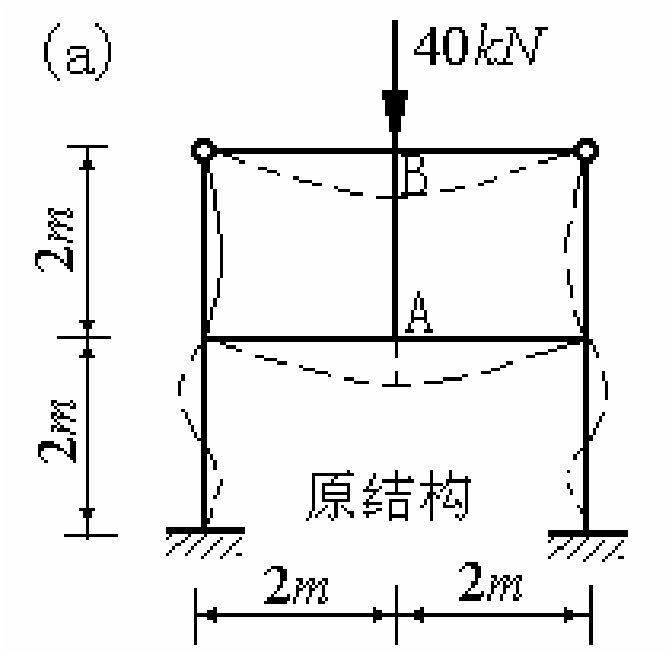

$$Z_1=-\frac{ql^2}{72i}$$



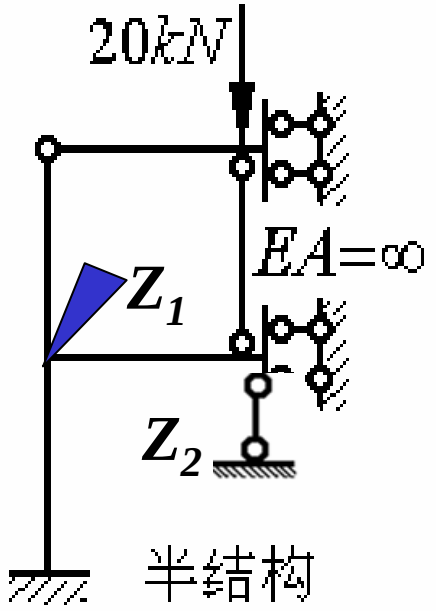

$M$ 图(kN·m)

# 例9

利用对称性简化图a所示的对称结构，取出最简的计算简图、基本体系，并作出*M*图。




*AB*杆不会弯曲而只受轴力。不计轴向变形影响，故将*AB*杆看作轴向刚度无限的链杆，则*A*，*B*两点的竖向位移相同。

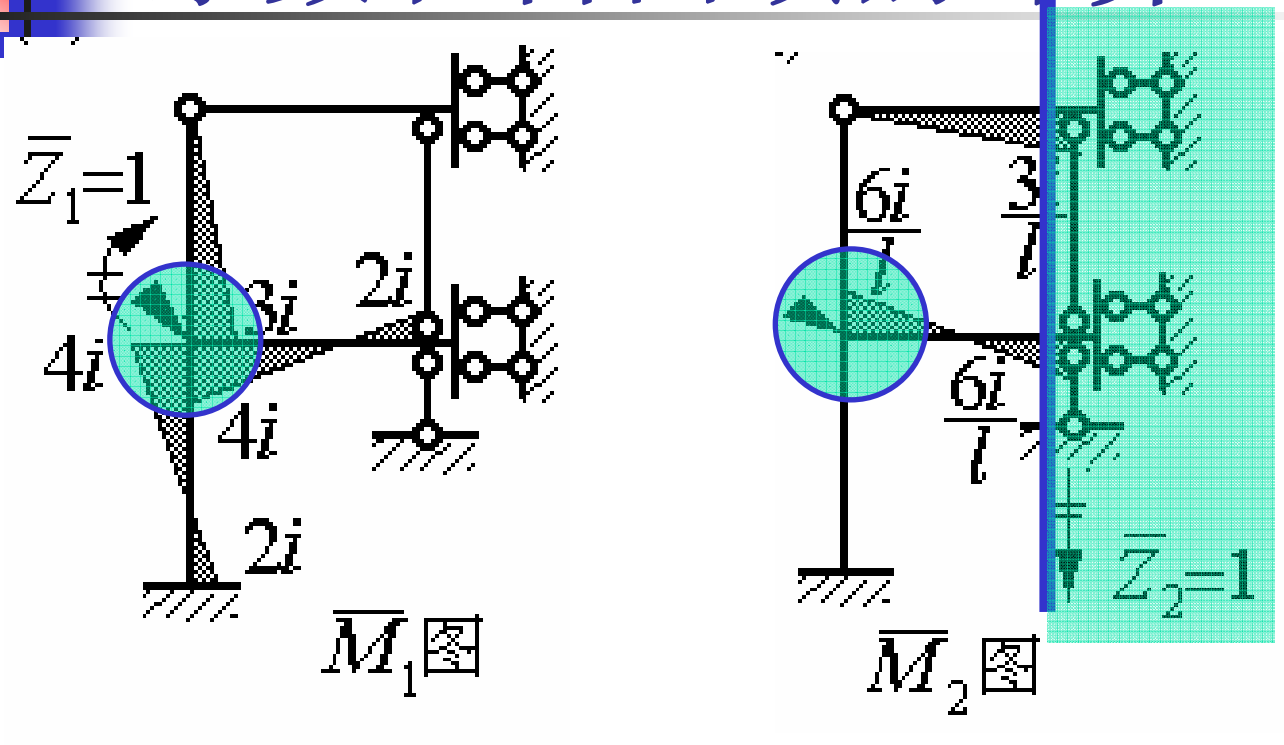



$$r_{11}Z_1 + r_{12}Z_2 + R_{1P} = 0$$

$$r_{21}Z_1 + r_{22}Z_2 + R_{2P} = 0$$

# 系数和自由项的计算





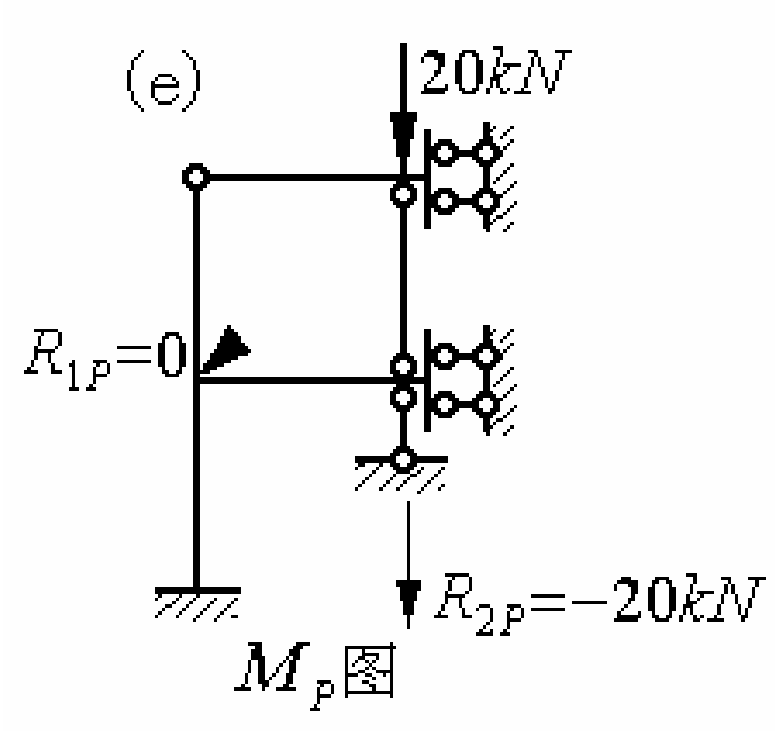

$$r_{11}=3i+4i+4i=11i$$

$$r_{22}=\frac{3i}{l^2}+\frac{12i}{l^2}=\frac{15i}{l^2}$$

$$r_{12}=r_{21}=-\frac{6i}{l}$$

# 系数和自由项的计算





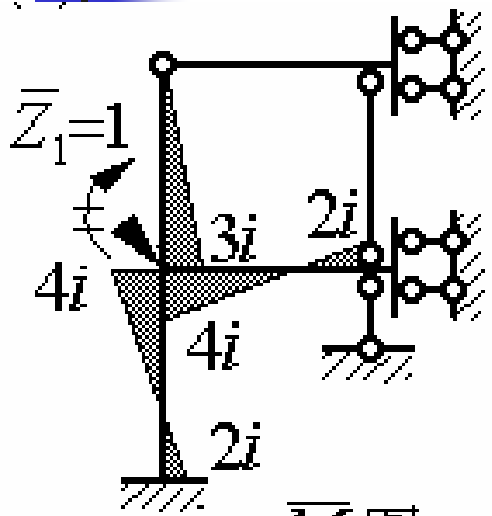

$$R_{1P}=0 \qquad R_{2P}=-20kN$$

$$Z_1=\frac{80}{43i}$$

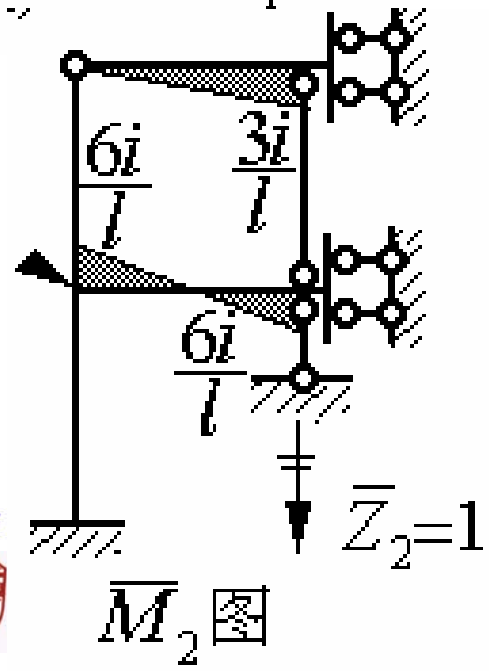
$$Z_2=\frac{220l}{129i}$$

# 按叠加法 $M=\overline{M}_1Z_1+\overline{M}_2Z_2$



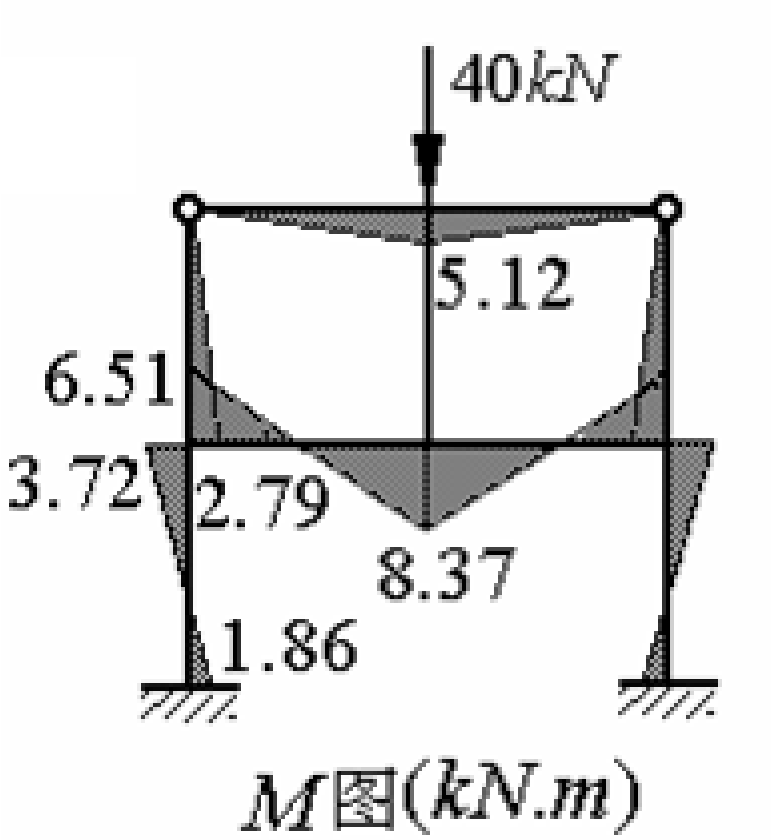



$\overline{M}_1$图




$\overline{M}_2$图

$$Z_1=\frac{80}{43i}\qquad Z_2=\frac{220l}{129i}$$




$M$图$(kN.m)$